Math 1 - Summer 2020 - July 9

Important Concepts

- Rational Functions
  - Vertical asymptotes come from the denominator of the function
  - Horizontal asymptotes depend on degree of numerator and denominator
- Factoring polynomials
  - The possible rational zeros of a polynomial $p(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$ are

$$\frac{\text{factors of } a_0}{\text{factors of } a_n}$$

Horizontal Asymptotes of a Rational Function

$$f(x) = \frac{\text{polynomial } p(x)}{\text{polynomial } q(x)}$$

Case I. If $\deg p(x) < \deg q(x)$ then H.A. $y=0$

e.g. $\frac{1}{x}$ H.A. $y=0$

Case II. If $\deg p(x) = \deg q(x)$ then H.A. $y=$ ratio of leading coeff

e.g. $y = \frac{5x-2}{2x+1}$ H.A. $y = \frac{5}{2}$

Case III. If $\deg p(x) > \deg q(x)$ no H.A., oblique asymptote

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Example 1. Let $f(x) = \dfrac{2x^1 - 3}{x^1 + 4}$. Find the horizontal intercepts, vertical intercept, vertical asymptotes, and horizontal asymptote. Use this information to sketch the graph.

x-int: numerator

$2x - 3 = 0 \quad x = \frac{3}{2} \quad \left(\frac{3}{2}, 0\right)$

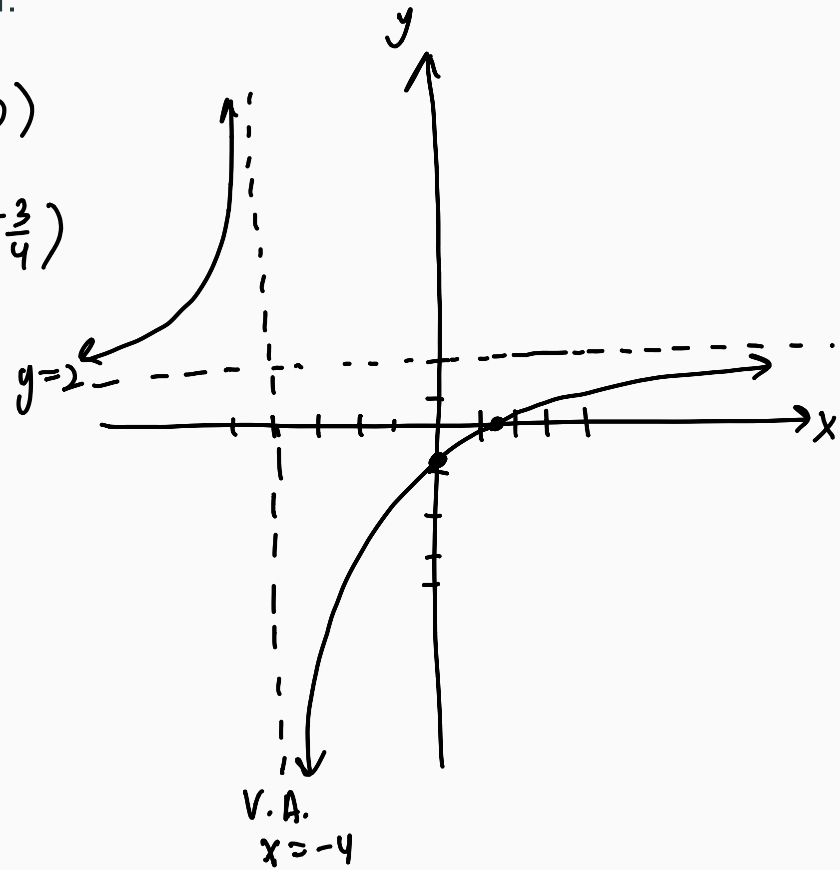
y-int: $f(0) = \dfrac{2(0)-3}{(0+4)} = -\dfrac{3}{4} \quad \left(0, -\frac{3}{4}\right)$

V.A. denominator

$x + 4 = 0 \quad x = -4$

H.A. deg top = 1 = deg bottom

$y = \dfrac{2}{1} = 2$



Example 2. Let $f(x) = \dfrac{x-5}{3x-1}$. Find the horizontal intercepts, vertical intercept, vertical asymptotes, and horizontal asymptote. Use this information to sketch the graph.

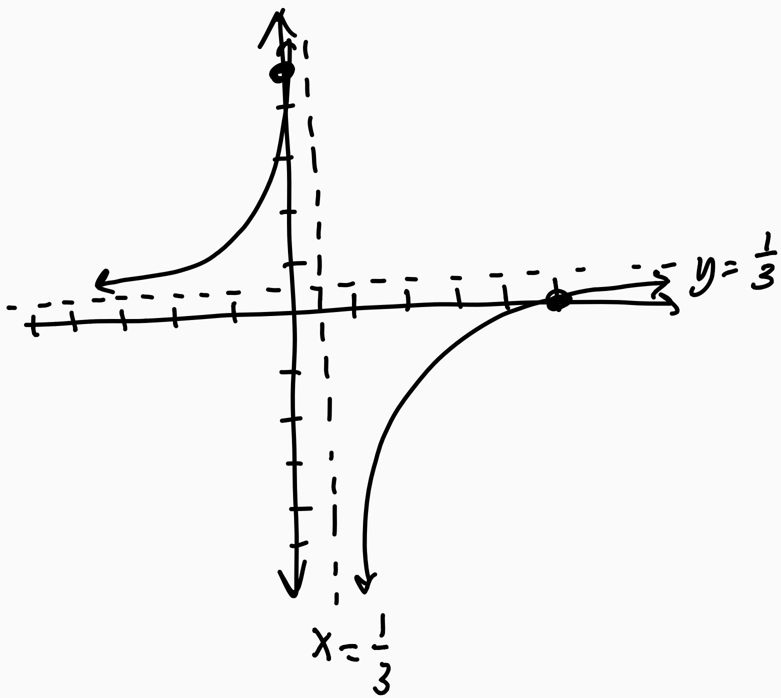
Try on your own for a couple minutes

x-int: $x-5=0 \quad x=5 \quad (5,0)$

y-int: $f(0) = \dfrac{0-5}{3\cdot 0-1} = \dfrac{5}{1} \quad (0,5)$

V.A.: $3x-1=0 \quad x=\frac{1}{3}$

H.A.: $y=\frac{1}{3}$



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Example 3. Given $Q(x) = \dfrac{(x-1)^1(x+1)^1}{(x-2)^2(x+2)^1}$, answer the following:

(a) What are the coordinates of the x-intercepts?

$(x-1)(x+1) = 0$

$x-1=0$ or $x+1=0$

$x=1$ $\quad$ $x=-1$

$(1,0), (-1,0)$

(b) What are the coordinates of the Q-intercepts? $\dfrac{(0-1)(0+1)}{(0-2)^2(0+2)} = \dfrac{-1}{8}$ $\quad$ $(0, -\frac{1}{8})$

(c) What are the vertical asymptotes?

$(x-2)^2(x+2)=0$

$(x-2)^2=0$

$x=2 \leftarrow$ mult 2

$x+2=0$

$x=-2$

(d) What are the horizontal asymptotes? deg top $=2$ $\quad$ deg bottom $=3$ $\quad$ $y=0$

(e) As $x \to \infty$, $Q(x) \to$ __0__

(f) As $x \to -\infty$, $Q(x) \to$ __0__

(g) Sketch the graph of $Q(x)$ (next slide)

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Example 3. Given $Q(x) = \dfrac{(x-1)(x+1)}{(x-2)^2(x+2)}$, answer the following:

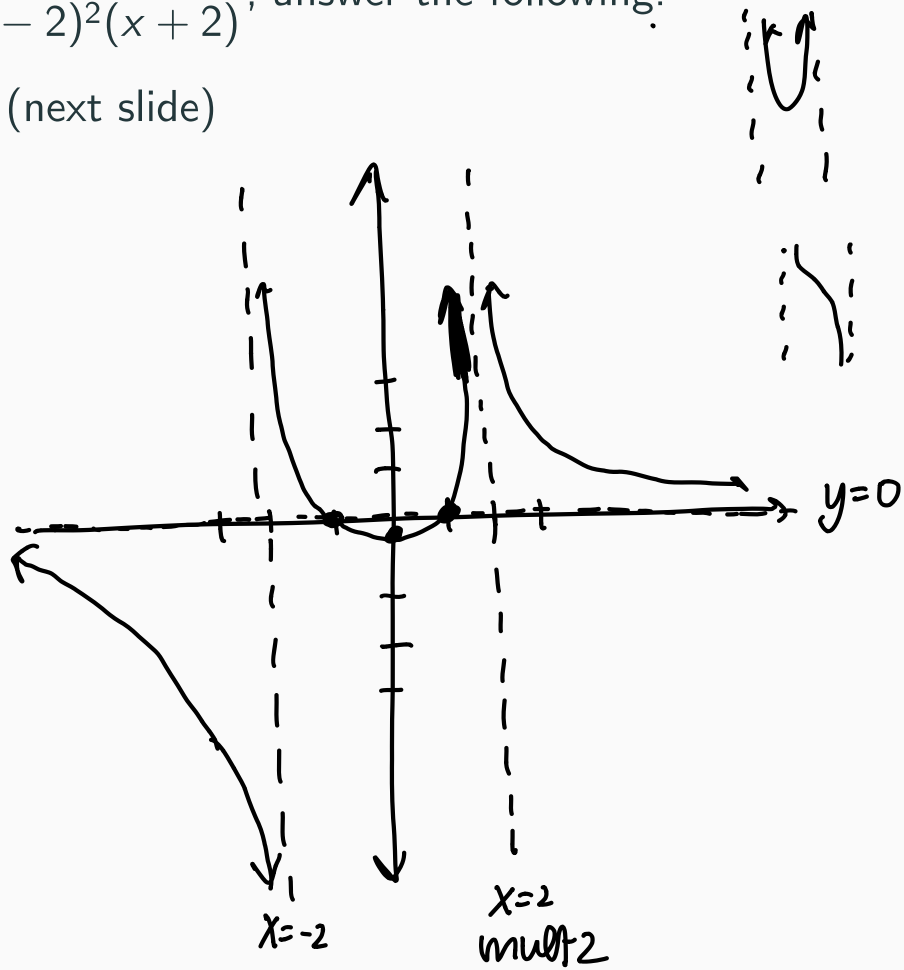
(g) Sketch the graph of $Q(x)$ (next slide)

x-int $(1,0)$ $(-1,0)$

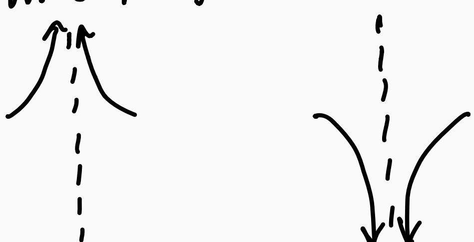
y-int $(0, -\frac{1}{8})$

V.A. $x=2$ mult 2 $\quad x=-2$

H.A. $y=0$



even multiplicity w/ V.A.



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Example 4. Given $Q(x) = \dfrac{3(x+3)^{1}(x-1)^{1}}{(x-2)^2}$, answer the following:

Try on your own for a few minutes

(a) What are the coordinates of the x-intercepts?

$(x+3)(x-1)=0$ $\quad$ $x+3=0$, $x=-3$ $\quad$ $x-1=0$, $x=1$ $\quad$ $(-3,0)$, $(1,0)$

(b) What are the coordinates of the Q-intercepts? $\dfrac{3(0+3)(0-1)}{(0-2)^2} = \dfrac{-9}{4}$ $\quad$ $(0, -\frac{9}{4})$

(c) What are the vertical asymptotes?

$(x-2)^2=0$

$x=2$ $\quad$ mult 2

(d) What are the horizontal asymptotes? deg top $=2$ $\quad$ deg bottom $=2$

$y = \dfrac{3}{1} = 3$

(e) As $x \to \infty$, $Q(x) \to$ $\underline{\ 3\ }$

(f) As $x \to -\infty$, $Q(x) \to$ $\underline{\ 3\ }$

(g) Sketch the graph of $Q(x)$ (next slide)

Example 4. Given $Q(x) = \frac{3(x+3)(x-1)}{(x-2)^2}$, answer the following:

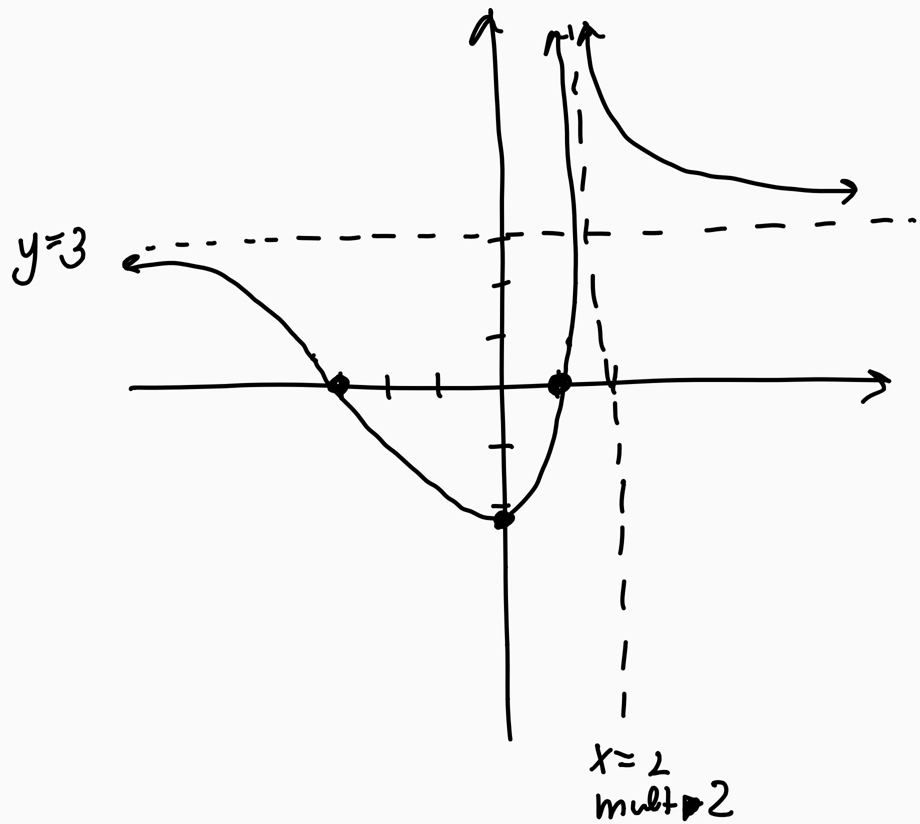
(g) Sketch the graph of $Q(x)$ (next slide)

x int: $(-3,0)$, $(1,0)$

y-int: $(0, -\frac{9}{4})$

V.A. $x=2$ mult 2

H.A. $y=3$



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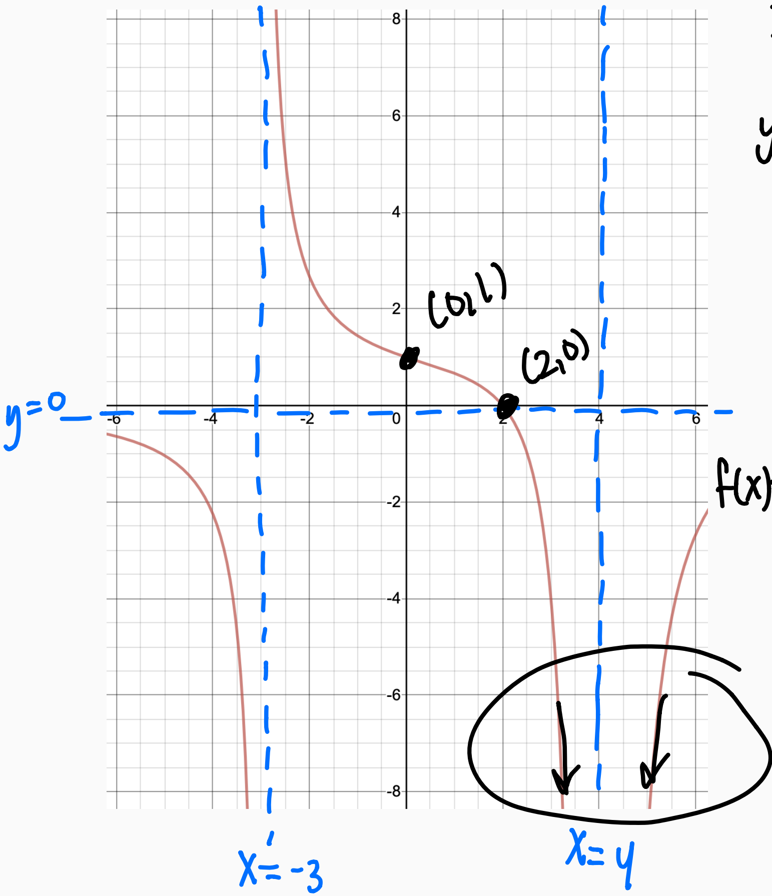
Example 5. Write the equation of the rational function whose graph is given below:



x-int (2,0)

y-int (0,1) ← to find a

V.A. $x=-3$, $x=4$ mult 2

H.A. $y=0$ ← deg of num < deg denom (check)

$$f(x) = \frac{a(x-2)}{(x+3)(x-4)^2}$$

$$\boxed{f(x) = \frac{-24(x-2)}{(x+3)(x-4)^2}}$$

$$\frac{a(0-2)}{(0+3)(0-4)^2} = 1$$

$$\frac{-2a}{48} = 1 \qquad a = -24$$

Example 6. Factor $p(x) = \underline{2}x^4 - 7x^3 + x^2 + 7x \underline{- 3}.$

possible zeros: $\dfrac{\text{factors of } 3}{\text{factors of } 2} = \dfrac{\pm 1, 3}{\pm 1, 2} = \pm 1, 3, \frac{1}{2}, \frac{3}{2}$

add

multiply

1 is a zero

$$\begin{array}{r|rrrrr|r} 1 & 2 & -7 & 1 & 7 & -3 & \\ & & 2 & -5 & -4 & 3 & \\ \hline & 2 & -5 & -4 & 3 & 0 & \end{array}$$

remainder

$(x-1)(2x^3-5x^2-4x+3)$

-1 is a zero

$(x-1)(x+1)(\overset{a}{2}x^2 \overset{b}{-7}x + \overset{c}{3})$

$$\begin{array}{r|rrrr|r} -1 & 2 & -5 & -4 & 3 & \\ & & -2 & 7 & -3 & \\ \hline & 2 & -7 & 3 & 0 & \end{array}$$

$$x = \frac{7 \pm \sqrt{49-4(2)(3)}}{2(2)} = \frac{7 \pm \sqrt{25}}{4} = \frac{7 \pm 5}{4}$$

$$= 3, \frac{1}{2}$$

$$\boxed{(x-1)(x+1)(x-3)(2x-1)}$$

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Example 7. Factor $p(x) = x^3 - 2x^2 - 4x + 8$.

possible zeros $= \dfrac{\text{factors of } 8}{\text{factors of } 1} = \dfrac{\pm 1, 2, 4, 8}{\pm 1} = \pm 1, 2, 4, 8$

$$\begin{array}{r|rrrr} 1 & 1 & -2 & -4 & 8 \\ & & 1 & -1 & -5 \\ \hline & 1 & -1 & -5 & \boxed{3} \end{array}$$

$$\begin{array}{r|rrrr} -1 & 1 & -2 & -4 & 8 \\ & & -1 & 3 & 1 \\ \hline & 1 & -3 & -1 & \boxed{9} \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & -2 & -4 & 8 \\ & & 2 & 0 & -8 \\ \hline & 1 & 0 & -4 & \boxed{0} \end{array}$$

2 is a zero!

$(x-2)(x^2+0x-4)$

$(x-2)(x^2-4)$

$(x-2)(x-2)(x+2)$

$\boxed{(x-2)^2(x+2)}$