

Important Concepts

- Exponential Functions
- Logarithmic Functions

$$a^x = y \quad \Leftrightarrow \quad \log_a y = x$$

$$a^{\log_a x} = x, \quad \log_a(a^x) = x$$

$$\log A + \log B = \log AB, \quad \log A - \log B = \log \left(\frac{A}{B} \right), \quad B \log A = \log A^B$$

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Example 1. Find a formula of an exponential function that passes through $(-2, 6)$ and $(3, 1)$.

$$f(x) = ab^x$$

$$(-2, 6) :$$

$$6 = ab^{-2} \quad \leftarrow \quad 6 = \frac{a}{b^2}$$

$$(3, 1) :$$

$$1 = ab^3$$

$$1 = 6b^2 b^3$$

$$1 = 6b^5$$

$$b^5 = \frac{1}{6}$$

$$b = \sqrt[5]{\frac{1}{6}}$$

$$a = 6b^2$$

$$a = 6\left(\sqrt[5]{\frac{1}{6}}\right)^2$$

$$f(x) = 6\left(\sqrt[5]{\frac{1}{6}}\right)^2 \left(\sqrt[5]{\frac{1}{6}}\right)^x$$

$$= \boxed{6\left(\sqrt[5]{\frac{1}{6}}\right)^{x+2}}$$

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Example 2. Find a formula of an exponential function that passes through $f(1) = 3$ and $f(-1) = 12$.

Try as you can for a couple minutes

$$a = 6$$
$$b = \frac{1}{2}$$

$f(x) = ab^x$

$(1, 3)$

$(-1, 12)$

$$3 = ab^1$$
$$12 = ab^{-1} \leftarrow 12 = \frac{a}{b}$$
$$a = 12b$$

$$3 = 12bb$$

$$3 = 12b^2$$

$$\frac{1}{4} = b^2$$

$$b = \frac{1}{2}$$

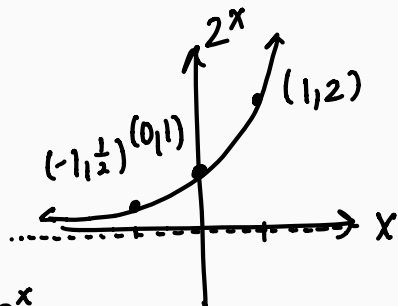
$$a = 12\left(\frac{1}{2}\right) = 6$$

$$f(x) = 6\left(\frac{1}{2}\right)^x$$

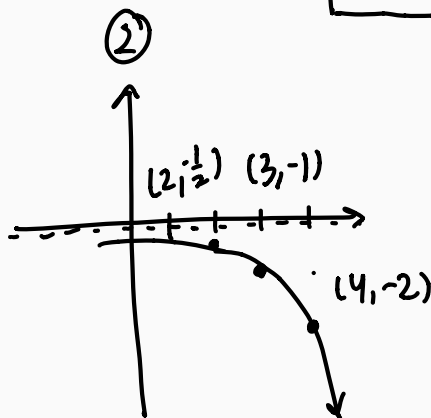
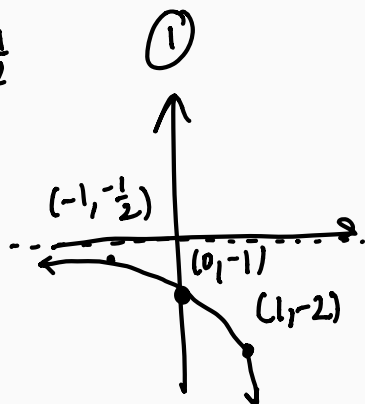
not 3^x

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Example 3. Sketch the graph of $f(x) = -2^{x-3} + 1$. What is the long run behavior as $x \rightarrow \infty, x \rightarrow -\infty$?

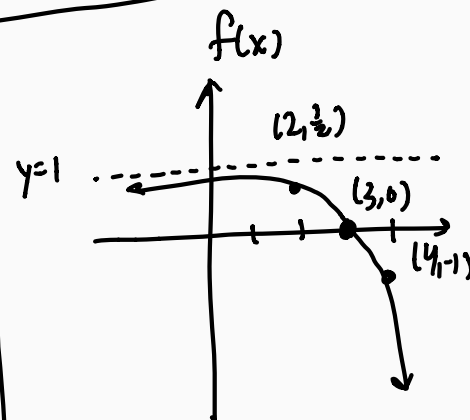


x	2^x
0	1
1	2
-1	$\frac{1}{2}$



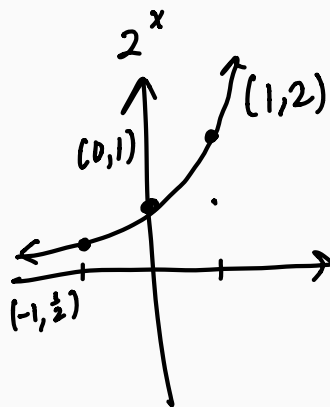
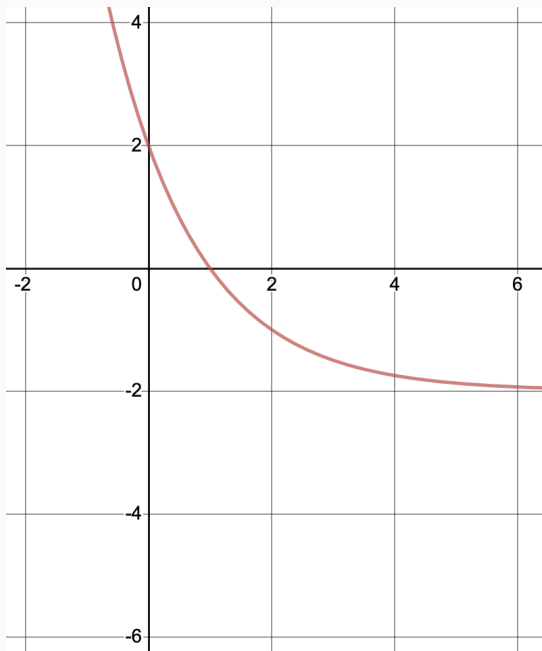
- read left to right
- ① vertical reflection
 - ② horizontal shift, right 3
 - ③ vertical shift, up 1

$$\begin{aligned} \text{As } x \rightarrow \infty, f(x) &\rightarrow -\infty \\ x \rightarrow -\infty, f(x) &\rightarrow 1 \end{aligned}$$



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Example 4. Find a formula for the following function graphed as a transformation of $f(x) = 2^x$.



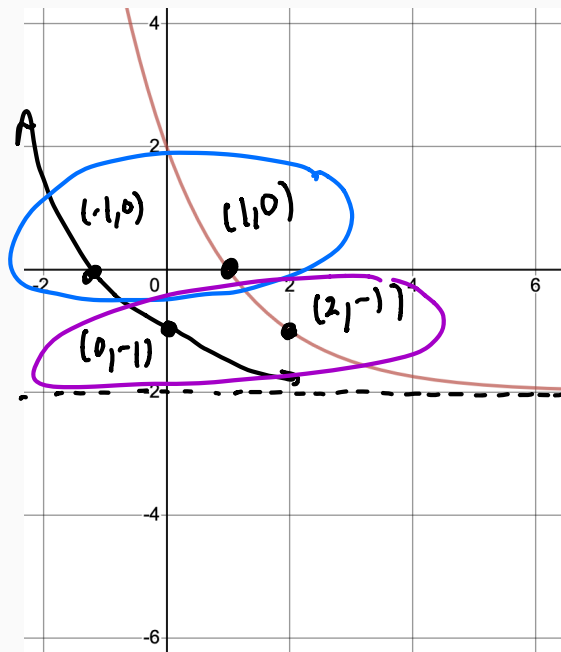
Hint: 3 transformations

Try to find the transformations as your own for a couple minutes

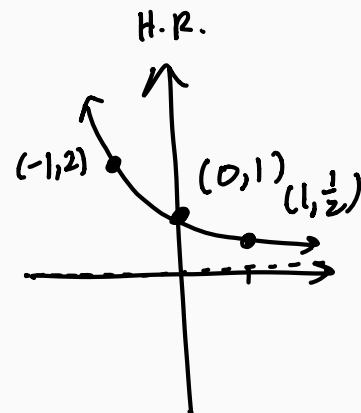
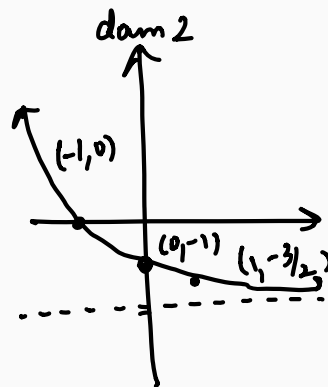
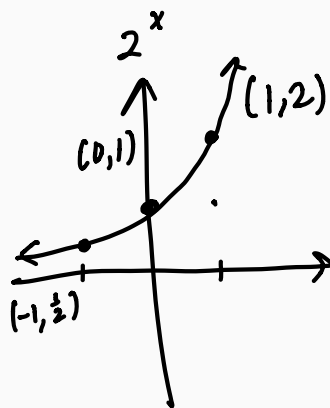
- ① Horizontal reflection
- ② Down 2
- ③ ?

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Example 4. Find a formula for the following function graphed as a transformation of $f(x) = 2^x$.



$$g(x) = 2^{-(x-2)} - 2$$



- Need to go right 2
- ① horizontal reflection
 - ② dum 2
 - ③ right 2

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Example 5. Simplify the following using logarithm properties:

(a) $\log_5(25)$

$$\log_5(5^2) = \textcircled{2}$$

(b) $\ln(e^3)$

$$\log_e(e^3) = \textcircled{3}$$

(c) $\log_6(\sqrt{6})$

$$\log_6(6^{1/2}) = \textcircled{\frac{1}{2}}$$

(d) $\log(0.001)$

$$\log_{10}(10^{-3}) = \textcircled{-3}$$

Try c & d on your own

$$\log_a(a^x) = x$$

$$\sqrt[m]{a} = a^{1/m}$$

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Example 6. Solve for x :

(a) $\log_3(x) = 2$

$$3^2 = x \rightarrow \textcircled{x=9}$$

(c) $7^x = \frac{1}{15}$

$$\log_7\left(\frac{1}{15}\right) = x$$

$$a^x = y \Leftrightarrow \log_a y = x$$

(b) $3^{4x-5} = 38$

$$\log_3 38 = 4x - 5$$

$$5 + \log_3 38 = 4x \rightarrow$$

$$\boxed{x = \frac{5 + \log_3 38}{4}}$$

(d) $\ln(2x + 3) = 1$

$$\log_e(2x + 3) = 1$$

$$e^1 = 2x + 3$$

$$2x = e - 3$$

$$\boxed{x = \frac{e-3}{2}}$$

Try c & d on your own for a couple minutes

Example 7. The population of Algeria was 34.9 million in 2009 and has been growing by 1.5% each year. If this trend continues, when will the population exceed 45 million?