Math 1 - Summer 2020 - July 14

Important Concepts

- Exponential Functions
- Logarithmic Functions

$$a^x = y \quad \Leftrightarrow \quad \log_a y = x$$

$$a^{\log_a x} = x, \quad \log_a(a^x) = x$$

$$\log A + \log B = \log AB, \quad \log A - \log B = \log\left(\frac{A}{B}\right), \quad B \log A = \log A^B$$

$$y = a(1+r)^x \quad \text{“Periodic growth”}$$

$$y = ae^{kx} \quad \text{“Continuous growth”}$$

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Another way to think of converting between logarithms and exponential functions

one way $\quad a^x = y \Leftrightarrow x = \log_a y$

Ex $\quad x^2 = 9$

$\sqrt{x^{\cancel{2}}} = \sqrt{9}$

$x = \pm 3$

Inverse properties:

$\log_a a^x = x, \quad a^{\log_a x} = x$

Ex $\quad 4^x = 7$

$\cancel{\log_4} 4^x = \log_4 7$

$\boxed{x = \log_4 7}$

Ex $\quad \log_2(x) = 7$

$\cancel{2}^{\cancel{\log_2}(x)} = 2^7$

$x = 2^7$

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Example 1. Solve for x.

(a) $5^x = 15$

$$\cancel{\log_5} 5^x = \log_5 15$$

$$\boxed{x = \log_5 15}$$

(b) $\log(3x + 1) = 1$

$$\cancel{10}^{\cancel{\log_{10}}(3x+1)} = 10^1$$

$$3x+1 = 10$$

$$3x = 9$$

$$x = 3$$

(c) $\log_2(x) = -3$

$$\cancel{2}^{\cancel{\log_2}(x)} = 2^{-3}$$

$$x = 2^{-3}$$

$$= \frac{1}{2^3}$$

$$= \frac{1}{8}$$

(d) $4^{2x-3} = 44$

Try c & d for a couple minutes

$$\cancel{\log_4} 4^{2x-3} = \log_4 44$$

$$2x-3 = \log_4 44$$

$$2x = 3 + \log_4 44$$

$$\boxed{x = \frac{3+\log_4 44}{2}}$$

Example 2. Simplify to a single logarithm using logarithm properties.

numerator denominator numerator Try on your own

(a) $\log(x) - \frac{1}{2}\log(y) + 3\log(z)$ (b) $2\log(x) + \frac{1}{3}\log(y) - \log(z)$

$\log x - \log(y^{1/2}) + \log(z^3)$

$\log\left(\frac{x}{y^{1/2}}\right) + \log(z^3)$

$\log\left(\frac{x}{y^{1/2}} \cdot \frac{z^3}{1}\right)$

$\boxed{\log\left(\frac{xz^3}{y^{1/2}}\right)}$

$\log\left(\frac{xz^3}{\sqrt{y}}\right)$

$\log(x^2) + \log(y^{1/3}) - \log z$

$\log(x^2 y^{1/3}) - \log z$

$\boxed{\log\left(\frac{x^2 y^{1/3}}{z}\right)}$

$\log A + \log B = \log(AB)$

$\log A - \log B = \log\left(\frac{A}{B}\right)$

$B \log A = \log(A^B)$

Example 3. Use logarithm properties to expand each expression.

Try b on your own for a bit

(a) $\log\left(\frac{x^{15}y^{13}}{z^{19}}\right)$

$\log(x^{15}y^{13}) - \log(z^{19})$

$\log(x^{15}) + \log(y^{13}) - \log(z^{19})$

$\boxed{15\log x + 13\log y - 19\log z}$

(b) $\log\left(\frac{x}{\sqrt{1-x^2}}\right)$

$\log x - \log\sqrt{1-x^2}$

$\log x - \log(1-x^2)^{1/2}$

$\boxed{\log x - \frac{1}{2}\log(1-x^2)}$

$\log A + \log B = \log(AB)$

$\log A - \log B = \log\left(\frac{A}{B}\right)$

$B\log A = \log A^B$

Example 4. The population of Algeria was 34.9 million in 2009 and has been growing by 1.5% each year. If this trend continues, when will the population exceed 45 million?

$y = a(1+r)^x$

initial amount

rate

$x =$ years since 2009

$a = 34.9$ (millions)

$r = 0.015$

Step 1. Find the formula

$P(x) = 34.9(1+0.015)^x$

$P(x) = 34.9(1.015)^x$

Step 2. Answer the question

set $P(x) = 45$

$45 = 34.9(1.015)^x$

$(1.015)^x = \frac{45}{34.9}$

use ln or log

$\ln(1.015)^x = \ln\left(\frac{45}{34.9}\right)$

$x \ln(1.015) = \ln\left(\frac{45}{34.9}\right)$

$x = \frac{\ln\left(\frac{45}{34.9}\right)}{\ln(1.015)} = 17.0718\ldots$

$\boxed{2026}$

Example 5. If \$1000 is invested in an account earning 3% compounded monthly, how long will it take the account to grow in value to \$1500?

$y = a(1+r)^x$

$x =$ months
$a = 1000$
$r = 0.03$

Try on your own for a few minutes

$P(x) = 1000(1.03)^x$

$1500 = 1000(1.03)^x$

$(1.03)^x = \frac{15}{10}$

$\ln(1.03)^x = \ln\left(\frac{15}{10}\right)$

$x \ln(1.03) = \ln\left(\frac{15}{10}\right)$

$x = \frac{\ln(15/10)}{\ln(1.03)} = 13.7\ldots$

$\boxed{\text{between 13 and 14 months}}$

Example 6. A scientist begins with 100 mg of a radioactive substance. After 6 days it has decayed to 60 mg. How long after the process began will it take to decay to 10 mg?

$$y = ae^{kx}$$

$a = 100$
$x =$ days

$$P(x) = 100e^{kx}$$

$x = 6 \qquad P(x) = 60$

$$60 = 100e^{6k}$$ solve for k

$$\ln\left(e^{6k}\right) = \ln\left(\frac{60}{100}\right)$$

$$6k = \ln\left(\frac{6}{10}\right)$$

$$k = \frac{\ln\left(\frac{6}{10}\right)}{6}$$

$$P(x) = 100e^{\frac{\ln\left(\frac{6}{10}\right)}{6}x}$$

$P(x) = 10$

$$10 = 100e^{\frac{\ln(6/10)}{6}x}$$

$$\ln\left(e^{\frac{\ln(6/10)}{6}x}\right) = \ln\left(\frac{1}{10}\right)$$

$$\frac{\ln(6/10)}{6}x = \ln\left(\frac{1}{10}\right)$$

$$x = \frac{6\ln\left(\frac{1}{10}\right)}{\ln(6/10)} = 27.045\ldots$$

$\boxed{\text{between 27 \& 28 days}}$

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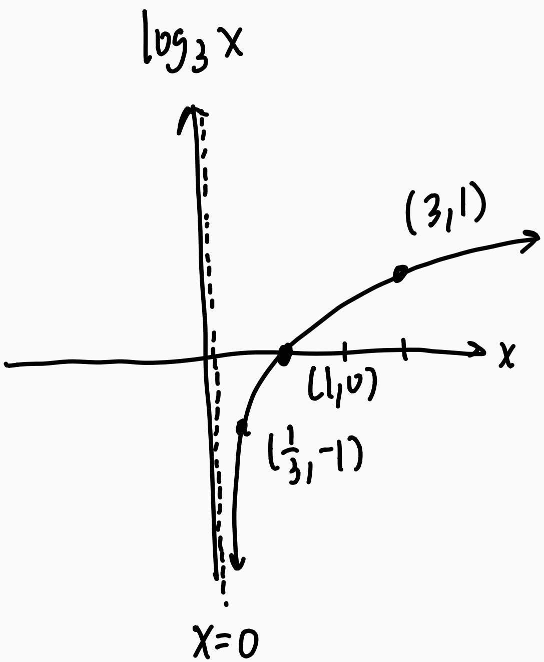
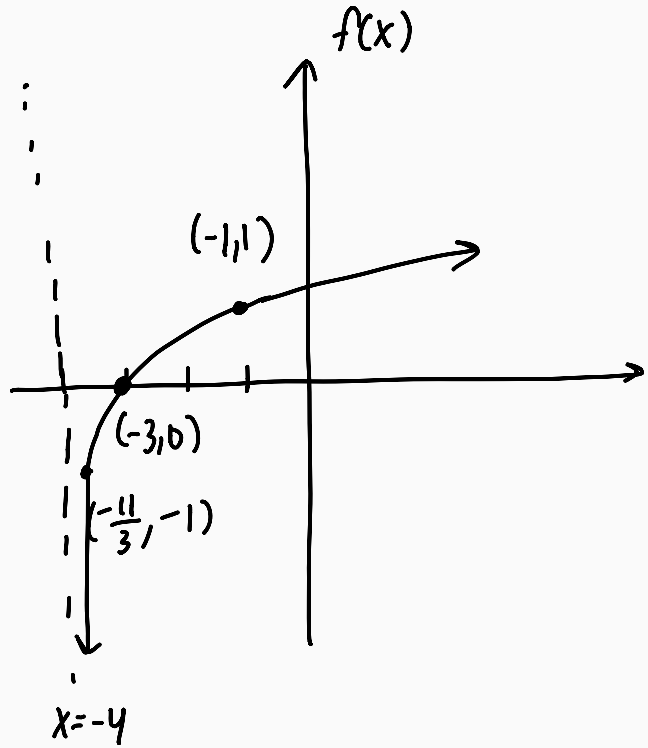
Example 7. Sketch the graph of $f(x) = \log_3(x \underline{+ 4})$ using transformations.

left 4 units





Example 8. Sketch the graph of $f(x) = \log_2(-x+1)$ using transformations.

rewrite: $\log_2(-(x-1))$

horizontal reflection

right 1