Math 1 - Summer 2020 - July 2

Important Concepts

- Inverse Functions
- Equations and inequalities involving absolute values
  - The equation $|x| = c$ can be split into two equations:

$$x = c, \quad x = -c$$

  - The inequality $|x| > c$ can be split into two inequalities

$$x > c, \quad x < -c$$

- Quadratic equations
  - The **transformation form** of the quadratic equation $f(x) = ax^2 + bx + c$ is

$$f(x) = a(x - h)^2 + k$$

where $h = -\dfrac{b}{2a}, k = f\left(-\dfrac{b}{2a}\right)$.

Example 1. Let $f(x) = \dfrac{x+2}{2x+3}$.

$a^{-1} = \dfrac{1}{a}$

(a) What is $f(0)$? $f(0) = \dfrac{0+2}{2 \cdot 0 + 3} = \dfrac{2}{3}$

(b) What is $(f(0))^{-1}$? ← this does not mean inverse function

$(f(0))^{-1} = \left(\dfrac{2}{3}\right)^{-1} = \dfrac{1}{\frac{2}{3}} = 1 \cdot \dfrac{3}{2} = \dfrac{3}{2}$

(c) Without finding $f^{-1}(x)$, what is $f^{-1}\left(\frac{2}{3}\right)$? ← this means inverse function

$f(x)$ $\qquad$ $f^{-1}(x)$

$(x, y) \longleftrightarrow (y, x)$

$f(x) = y$ $\qquad$ $f^{-1}(y) = x$

$f(0) = \dfrac{2}{3}$

$f^{-1}\left(\dfrac{2}{3}\right) = 0$

Example 2. Let $f(x) = \dfrac{x+2}{2x+3}$. Find $f^{-1}(x)$.

Step 1 Replace $f(x)$ w/ y

$$y = \frac{x+2}{2x+3}$$

Step 2 Swap x & y values

$$x = \frac{y+2}{2y+3}$$

Step 3 Solve for y

$$(2y+3)\,x = \frac{y+2}{2y+3}\,(2y+3)$$

$$2xy + 3x = y + 2$$

$$2xy - y = -3x + 2$$

$$y(2x-1) = -3x+2$$

$$y = \frac{-3x+2}{2x-1}$$

Step 4 Replace y w/ $f^{-1}(x)$

$$\boxed{f^{-1}(x) = \frac{-3x+2}{2x-1}}$$

Example 3. Let $f(x) = 3x - 5$. Find $f^{-1}(x)$. Sketch the graphs of $f(x)$ and $f^{-1}(x)$. About what line are $f(x)$ and $f^{-1}(x)$ symmetric?

Try this example on your own for a couple minutes.

Step 1 Replace $f(x)$ w/ y

$y = 3x - 5$

Step 2 Swap x & y

$x = 3y - 5$

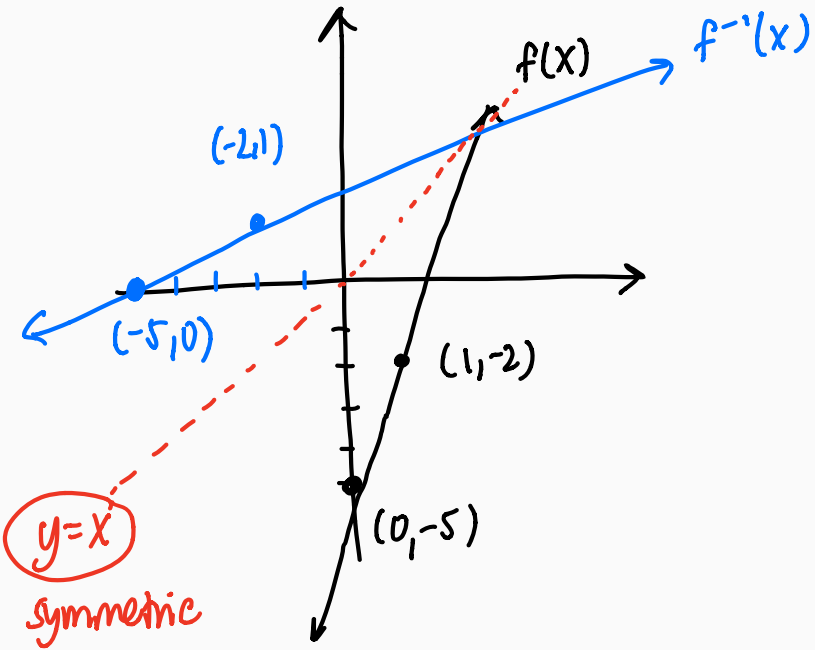
Step 3 Solve for y

$$\frac{x+5}{3} = \frac{3y}{3}$$

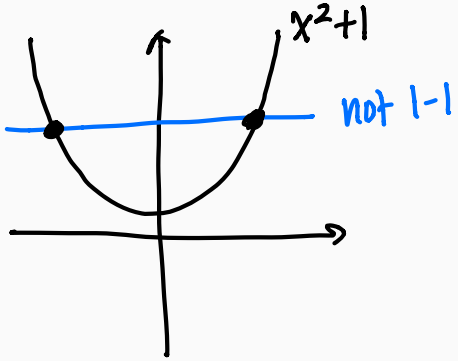
$$\frac{x+5}{3} = y$$

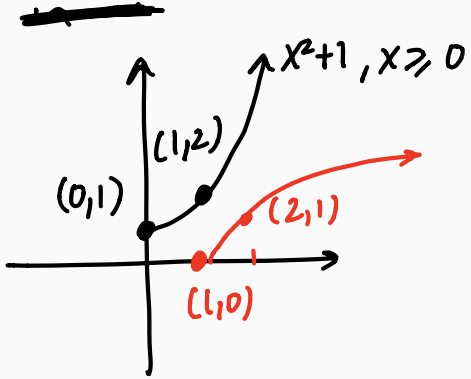
Step 4 Replace y w/ $f^{-1}(x)$

$$f^{-1}(x) = \frac{x+5}{3}$$



Example 4. Let $f(x) = x^2 + 1$, $x \geq 0$. Find $f^{-1}(x)$.





Step 1 $y = x^2 + 1$

Step 2 $x = y^2 + 1$

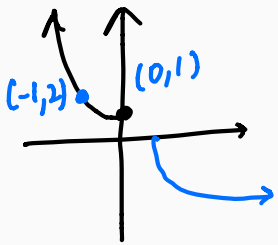
Step 3 $y^2 = x - 1$

$y = \pm\sqrt{x-1}$

Step 4 $\boxed{f^{-1}(x) = \sqrt{x-1}}$

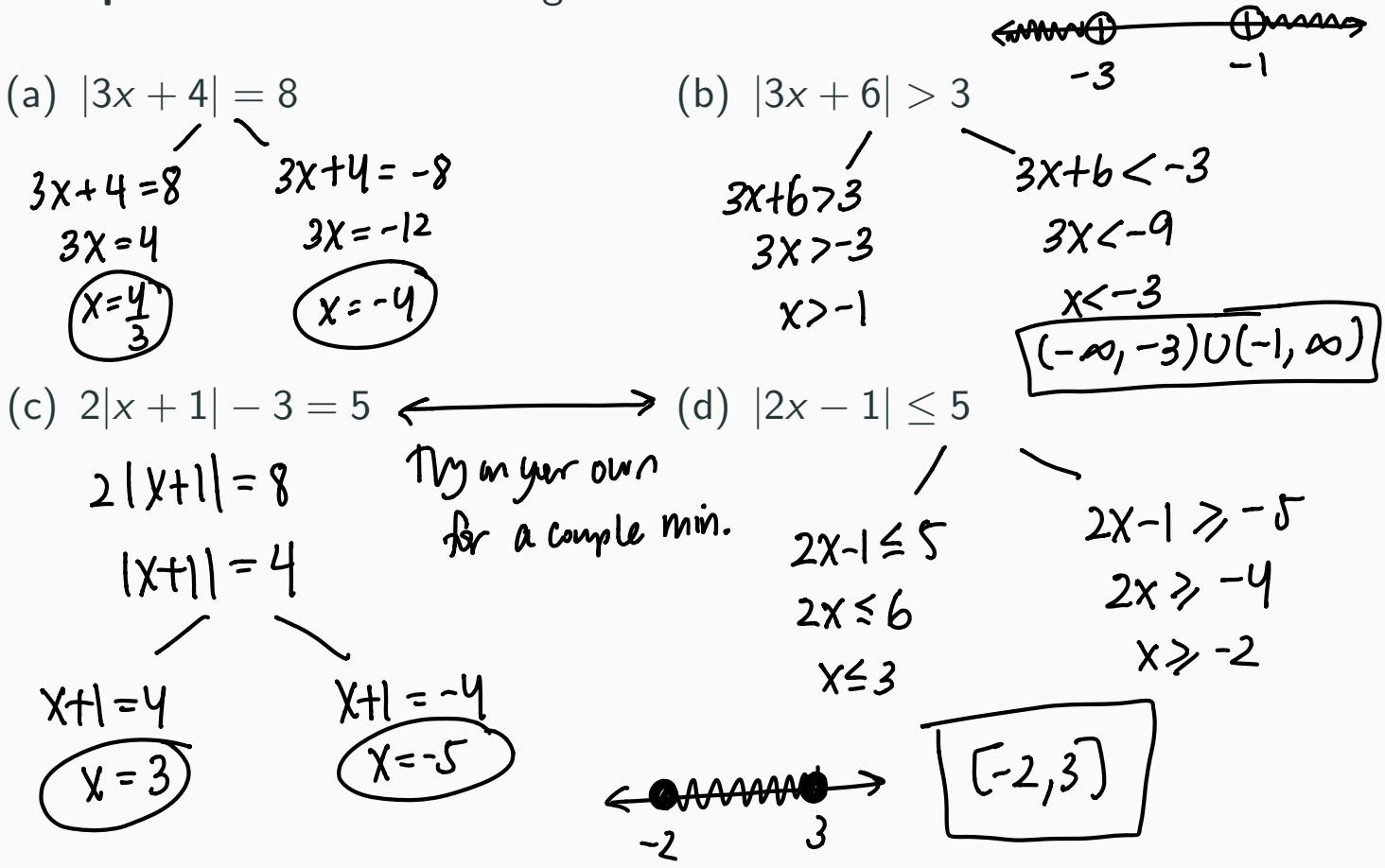
$f(x) = x^2 + 1$, $x \leq 0$ sometimes

$y = \pm\sqrt{x-1}$



$f^{-1}(x) = -\sqrt{x-1}$

Example 5. Solve the following:



Example 6. Write $f(x) = x^2 - 4x - 5$ in transformation form. What is the vertex of f? What are the intercepts of f? Sketch the graph of f.

$f(x) = ax^2 + bx + c$

Transformation form:

$f(x) = a(x-h)^2 + k$

$h = -\frac{b}{2a}$ $\quad k = f\left(-\frac{b}{2a}\right)$

vertex: (h,k)

$f(x) = 1x^2 - 4x - 5$

$a = 1 \quad b = -4$

$h = -\frac{b}{2a} = -\frac{(-4)}{2(1)} = \frac{4}{2} = 2$

$k = f(2) = 2^2 - 4(2) - 5 = 4 - 8 - 5 = -9$

$f(x) = 1(x-2)^2 - 9$

vertex: $(2, -9)$

x-int: $y = 0$

$x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

$x - 5 = 0 \quad x + 1 = 0$

$x = 5 \quad x = -1$

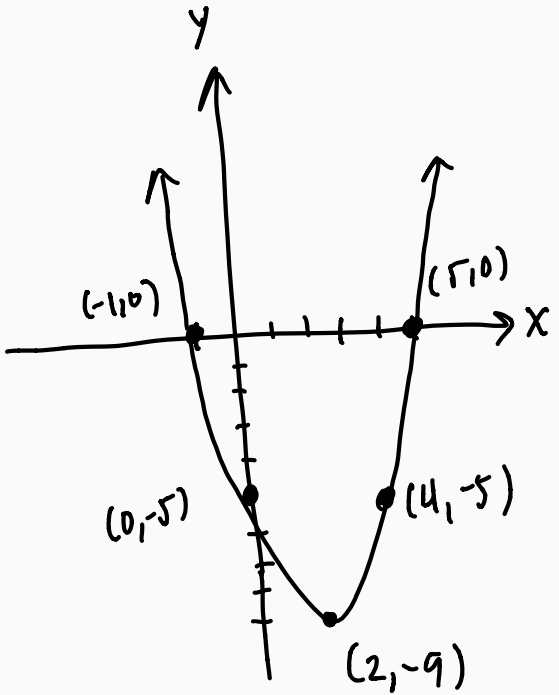
$(5, 0) \quad (-1, 0)$

y-int: $x = 0$

$f(0) = 0^2 - 4 \cdot 0 - 5 = -5$

$(0, -5)$

Example 6. Write $f(x) = x^2 - 4x - 5$ in transformation form. What is the vertex of f? What are the intercepts of f? Sketch the graph of f.



$f(x) = 1(x-2)^2 - 9$

vertex: $(2,-9)$

x-int: $y = 0$

$x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

$x - 5 = 0$ $\quad$ $x + 1 = 0$

$x = 5$ $\quad$ $x = -1$

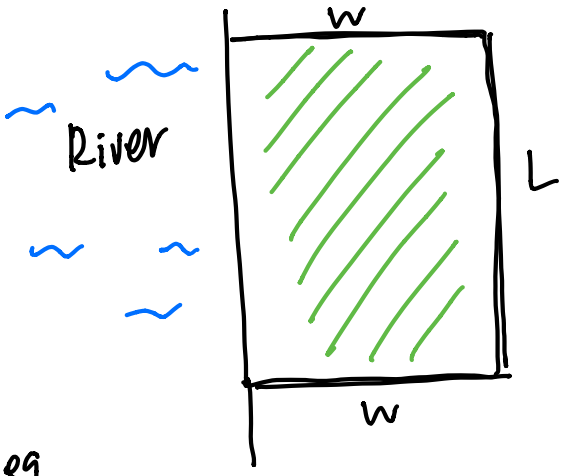
$(5,0)$ $\quad$ $(-1,0)$

y-int: $x = 0$

$f(0) = 0^2 - 4 \cdot 0 - 5 = -5$

$(0,-5)$

Ex A farmer wants to fence a rectangular area. The farmer has 1000 ft of fencing. What dimensions are needed to maximize the area?



$A = LW$

$P = 2W + L \rightarrow 1000 = 2W + L$

$L = 1000 - 2W$

$L = 500$ ft

① Area eq perimeter eq

② Narrow area eq down to 1 variable

③ Vertex of area

$A = (1000 - 2W)W$

$A = -2W^2 + 1000W$ quadratic equation

$h = \frac{-b}{2a} = \frac{-1000}{2(-2)} = \frac{-1000}{-4} = 250$ ← width

$W = 250$ ft

Ex Convert to transformation form:

$f(x) = -2x^2 + 16x + 7$

① $a = -2 \quad b = 16$

② $h = \frac{-b}{2a} = \frac{-16}{2(-2)} = \frac{-16}{-4} = 4$

③ $k = f(4) = -2(4)^2 + 16(4) + 7$

$= -32 + 64 + 7$

$= 32 + 7$

$= 39$

④ $f(x) = -2(x - 4)^2 + 39$