Math 1 - Summer 2020 - July 23

Important Concepts

- Distance between two points (x_1, y_1) and (x_2, y_2):

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- Equation of a circle with center (h, k) and radius r:

$$(x - h)^2 + (y - k)^2 = r^2$$

- Multiply degrees by $\dfrac{\pi}{180^\circ}$ to get radians
- Multiply radians by $\dfrac{180^\circ}{\pi}$ to get degrees
- To get coterminal angle add or subtract by $360^\circ(2\pi)$ until you get an angle between $0^\circ(0 \text{ rad})$ and $360^\circ(2\pi \text{ rad})$

Math 1 - Summer 2020 - July 23

Important Concepts

- Arc length: $s = r\theta$ (θ in radians)
- Area of sector of circle: $\boxed{A = \frac{1}{2}\theta r^2}$
- If (x, y) is a point on a circle with radius r with angle θ, then

$$x = r\cos\theta \quad \text{and} \quad y = r\sin\theta$$

- Pythagorean theorem: $a^2 + b^2 = c^2$
- Relationships between trigonometric functions:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}, \quad \csc\theta = \frac{1}{\sin\theta}, \quad \sec\theta = \frac{1}{\cos\theta}, \quad \cot\theta = \frac{1}{\tan\theta}$$

- Pythagorean identity:

$$\sin^2\theta + \cos^2\theta = 1$$

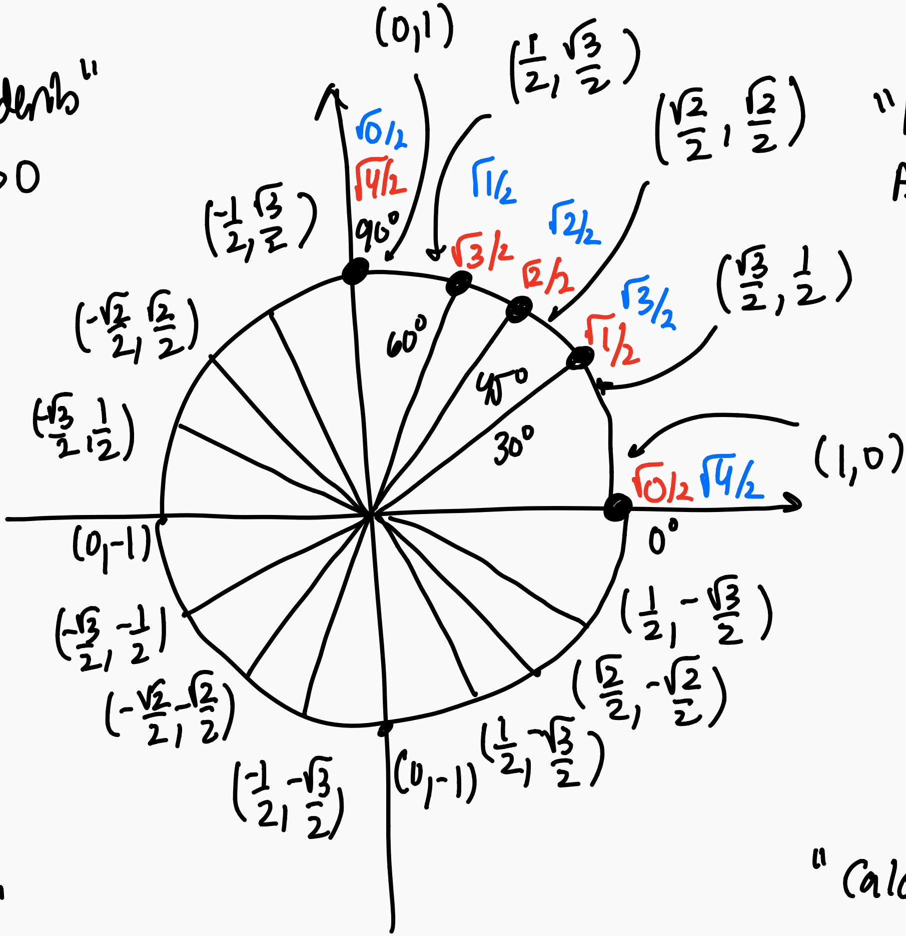
Unit circle

$y = \sin\theta$

$x = \cos\theta$

"Students"
sin > 0

"All"
All trig > 0

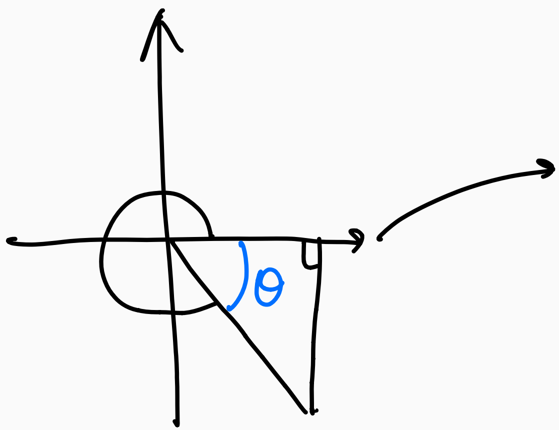


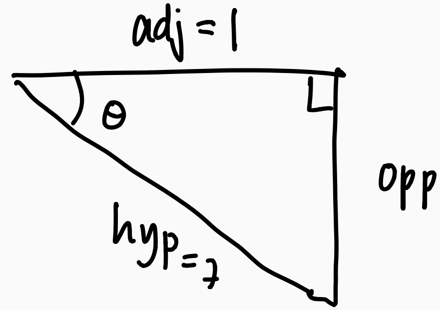
"Take"
tan > 0

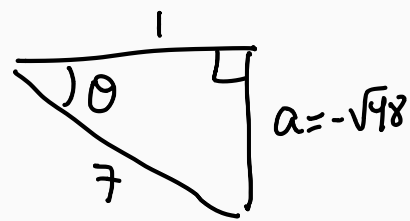
"Calculus"
cos > 0

Math 1 - Summer 2020 - July 23

Example 1. If $\cos\theta = \dfrac{1}{7}$ and θ is in the 4th quadrant, find $\sin\theta$.







Pythagorean's Thm

$1^2 + a^2 = 7^2$

$1 + a^2 = 49$

$a^2 = 48$

$a = \pm\sqrt{48}$

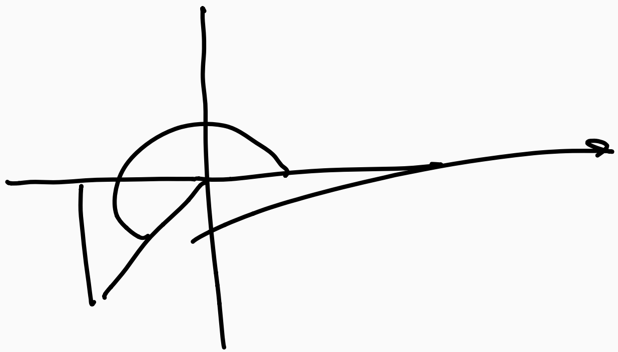
$a = -\sqrt{48}$

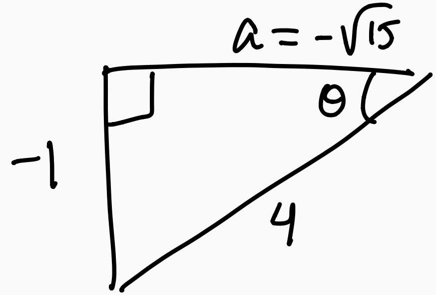
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{1}{7}$$

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{-\sqrt{48}}{7}$$

Example 2. If $\sin\theta = -\dfrac{1}{4}$ and θ is in the 3rd quadrant, find $\cos\theta$.

Try this on your own for a couple minutes





$$\sin\theta = -\frac{1}{4} = \frac{\text{opp}}{\text{hyp}}$$

$$a^2 + (-1)^2 = 4^2$$

$$a^2 + 1^2 = 16$$

$$a^2 = 15$$

$$a = \pm\sqrt{15}$$

$$a = -\sqrt{15}$$

$$\cos\theta = -\frac{\sqrt{15}}{4}$$

Example 3. Find the coordinates of the point on a circle with radius 15 corresponding to an angle of 135°.

Unit circle

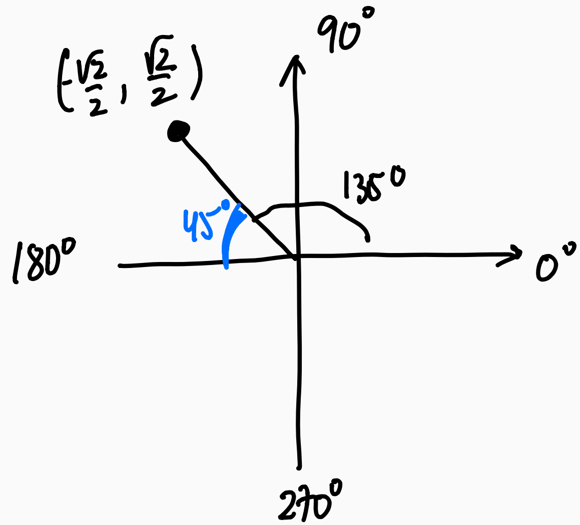
$x = \cos\theta$

$y = \sin\theta$

circle w/ radius r

$x = r\cos\theta$

$y = r\sin\theta$



$$x = 15\cos(135°) = 15\left(-\frac{\sqrt{2}}{2}\right) = \frac{-15\sqrt{2}}{2}$$

$$y = 15\sin(135°) = 15\left(\frac{\sqrt{2}}{2}\right) = \frac{15\sqrt{2}}{2}$$

reference angle : $180° - 135°$
$= 45°$

$$\boxed{\left(-\frac{15\sqrt{2}}{2}, \frac{15\sqrt{2}}{2}\right)}$$

Math 1 - Summer 2020 - July 23

Example 4. Find the coordinates of the point on a circle with radius 20 corresponding to an angle of 240°.

Try on your own for a couple minutes

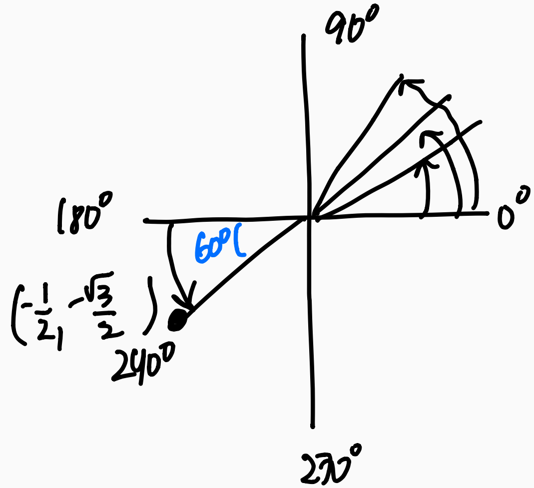
$x = r\cos\theta$

$y = r\sin\theta$

$x = 20\cos(240^\circ) = 20\left(-\frac{1}{2}\right) = -10$

$y = 20\sin(240^\circ) = 20\left(-\frac{\sqrt{3}}{2}\right) = -10\sqrt{3}$

$\boxed{(-10, -10\sqrt{3})}$

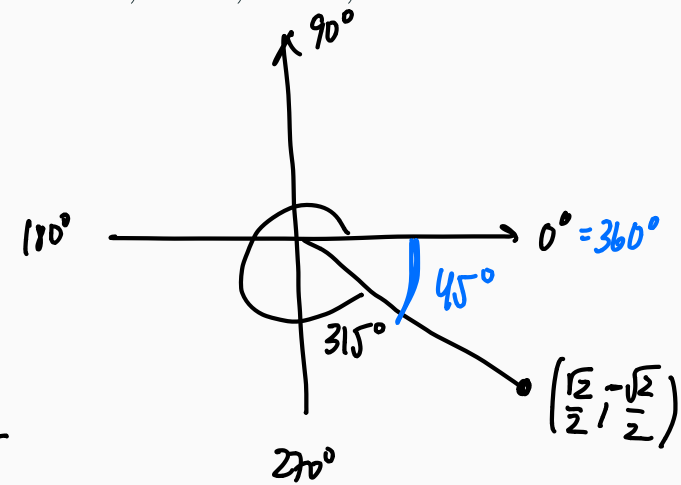


$240^\circ - 180^\circ = 60^\circ$

* reference angle is always measured from x-axis

Example 5. If $\theta = \dfrac{7\pi}{4}$, find exact values for $\sec\theta, \csc\theta, \tan\theta, \cot\theta$.

$$\frac{7\cancel{\pi}}{4}\left(\frac{\overset{45^\circ}{\cancel{180^\circ}}}{\cancel{\pi}}\right) = 7(45^\circ) = 315^\circ$$



$$360^\circ - 315^\circ = 45^\circ$$

$$\sin(315^\circ) = -\frac{\sqrt{2}}{2}$$

$$\cos(315^\circ) = \frac{\sqrt{2}}{2}$$

$$\frac{\sqrt{2}\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}}$$

$$\sec(315^\circ) = \frac{1}{\cos(315^\circ)} = \frac{1}{\frac{\sqrt{2}}{2}} = 1\cdot\frac{2}{\sqrt{2}} = \sqrt{2}$$

$$\csc(315^\circ) = \frac{1}{\sin(315^\circ)} = \frac{1}{-\frac{\sqrt{2}}{2}} = 1\cdot\frac{2}{-\sqrt{2}} = -\sqrt{2}$$

$$\tan(315^\circ) = \frac{\sin(315^\circ)}{\cos(315^\circ)} = \frac{-\sqrt{2}/2}{\sqrt{2}/2} = -1$$

$$\cot(315^\circ) = \frac{1}{\tan(315^\circ)} = \frac{1}{-1} = -1$$

Example 6. If $\theta = \dfrac{5\pi}{6}$, find exact values for $\sec\theta, \csc\theta, \tan\theta, \cot\theta$.

$\cos\theta = \frac{-\sqrt{3}}{2}$ $\quad \sin\theta = \frac{1}{2}$

$\sec\theta = \frac{1}{\cos\theta} = -\frac{2}{\sqrt{3}}$

$\csc\theta = \frac{1}{\sin\theta} = 2$

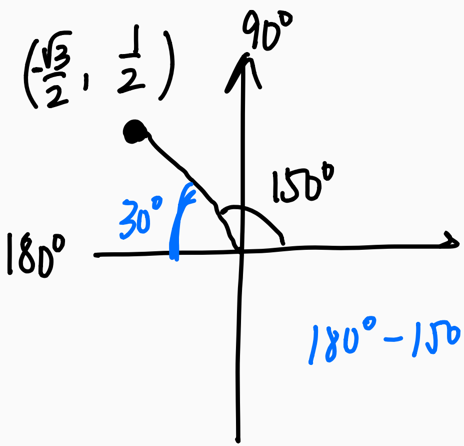
$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{1}{\cancel{2}} \cdot \frac{-\cancel{2}}{\sqrt{3}} = -\frac{1}{\sqrt{3}}$

$\cot\theta = \frac{1}{\tan\theta} = -\sqrt{3}$

Try on your own for a couple minutes

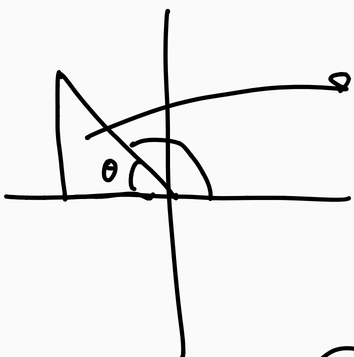
$\frac{5\pi}{6}$ in degrees is 150°

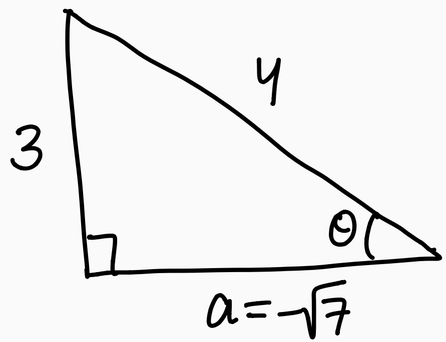
$\frac{5\cancel{\pi}}{\cancel{6}}\left(\frac{\cancel{180^\circ}^{\,30^\circ}}{\cancel{\pi}}\right) = 150^\circ$



$180^\circ - 150^\circ = 30^\circ$

Example 7. If $\sin\theta = \dfrac{3}{4}$ and θ is in quadrant II, find $\cos\theta$, $\sec\theta$, $\csc\theta$, $\tan\theta$, and $\cot\theta$.





$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{3}{4}$

Pythagorean:

$a^2 + 3^2 = 4^2$

$a^2 + 9 = 16$

$a^2 = 7$

$a = \pm\sqrt{7}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{-\sqrt{7}}{4}$

$\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{-4}{\sqrt{7}}$

$\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{4}{3}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{3}{-\sqrt{7}}$

$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{-\sqrt{7}}{3}$

Example 8. Simplify each of the following to an expression involving a single trig function with no fractions:

Goal: write everything in terms of sin & cos and then simplify

(a) $\csc(t)\tan(t) = \frac{1}{\cancel{\sin t}}\frac{\cancel{\sin t}}{\cos t} = \frac{1}{\cos t} = \sec t$

(b) $\dfrac{\sec(t)-\cos(t)}{\sin(t)}$ $\quad \dfrac{\frac{1}{\cos t} - \frac{\cos t}{1}\frac{\cos t}{\cos t}}{\sin t} = \dfrac{\frac{1-\cos^2 t}{\cos t}}{\frac{\sin t}{1}}$

$= \frac{1-\cos^2 t}{\cos t}\frac{1}{\sin t} = \frac{1-\cos^2 t}{\cos t \sin t} = \frac{\sin^{\cancel{2}} t}{\cos t \cancel{\sin t}} = \frac{\sin t}{\cos t} = \tan t$

$\boxed{* \ \sin^2 t + \cos^2 t = 1} \rightarrow 1-\cos^2 t = \sin^2 t$

$$\frac{1+\csc t}{1+\sin t} = \frac{\sin t}{\sin t} \frac{\frac{1}{1} + \frac{1}{\sin t}}{1+\sin t} = \frac{\frac{\sin t+1}{\sin t}}{\frac{1+\sin t}{1}} = \frac{\cancel{\sin t+1}}{\sin t} \cdot \frac{1}{\cancel{1+\sin t}}$$

$$= \frac{1}{\sin t} = \csc t$$