Math 1 - Summer 2020 - July 28

Announcements

- Thursday sections will be combined to an office hour / review session 12-2pm PT
- Thursday session will be recorded and posted on CCLE
- Bring your own questions for Thursday
    - Recommendation: bring textbook questions, `http://www.opentextbookstore.com/precalc/`
- NO office hour Thursday 2-3pm PT

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Important concepts

- $\sin(\theta)$ is an odd function, i.e. $\sin(-\theta) = -\sin(\theta)$
- $\cos\theta$ is an even function, i.e. $\cos(-\theta) = \cos(\theta)$
- $\sin^{-1}(\theta)$ has ~~domain~~ $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
- $\cos^{-1}(\theta)$ has ~~domain~~ $[0, \pi]$

range

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Important concepts

- More identities:

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$\cos(2\theta) = cos^2(\theta) - \sin^2(\theta) = 1 - 2\sin^2(\theta) = 2\cos^2(\theta) - 1$$

$$\sin^2\left(\frac{\theta}{2}\right) = \frac{1-\cos(\theta)}{2}, \quad \cos^2\left(\frac{\theta}{2}\right) = \frac{1+\cos(\theta)}{2}$$

$$\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \sin(\beta)\cos(\alpha)$$

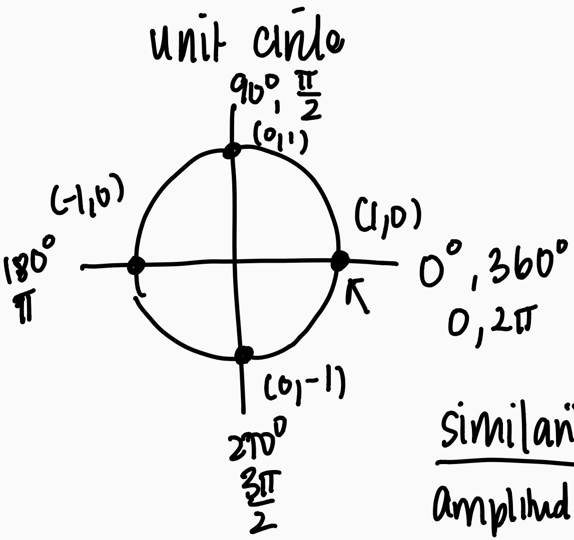
$$\sin(\alpha-\beta) = \sin(\alpha)\cos(\beta) - \sin(\beta)\cos(\alpha)$$

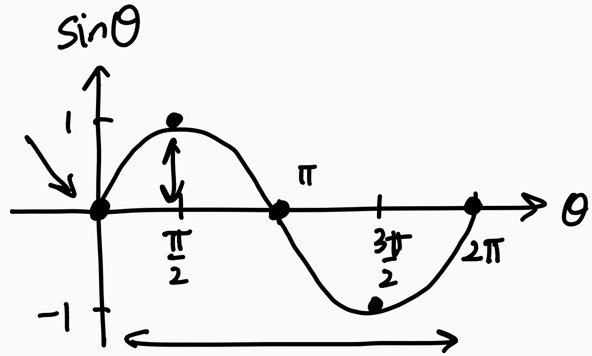
$$\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

$$\cos(\alpha-\beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$$

Graphs of sine and cosine

What are the similarities of the graphs of sine and cosine? What are differences?





$y = \sin\theta$

$x = \cos\theta$

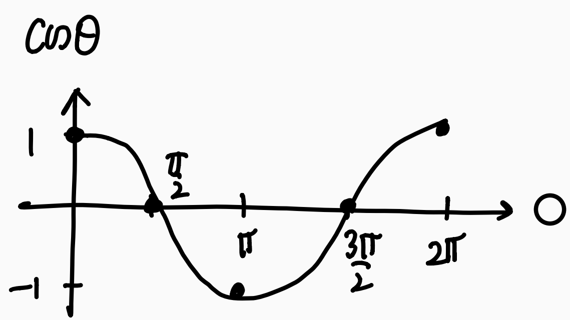
<u>similarities</u>

amplitude 1

period 2π

<u>differences</u>

starting value



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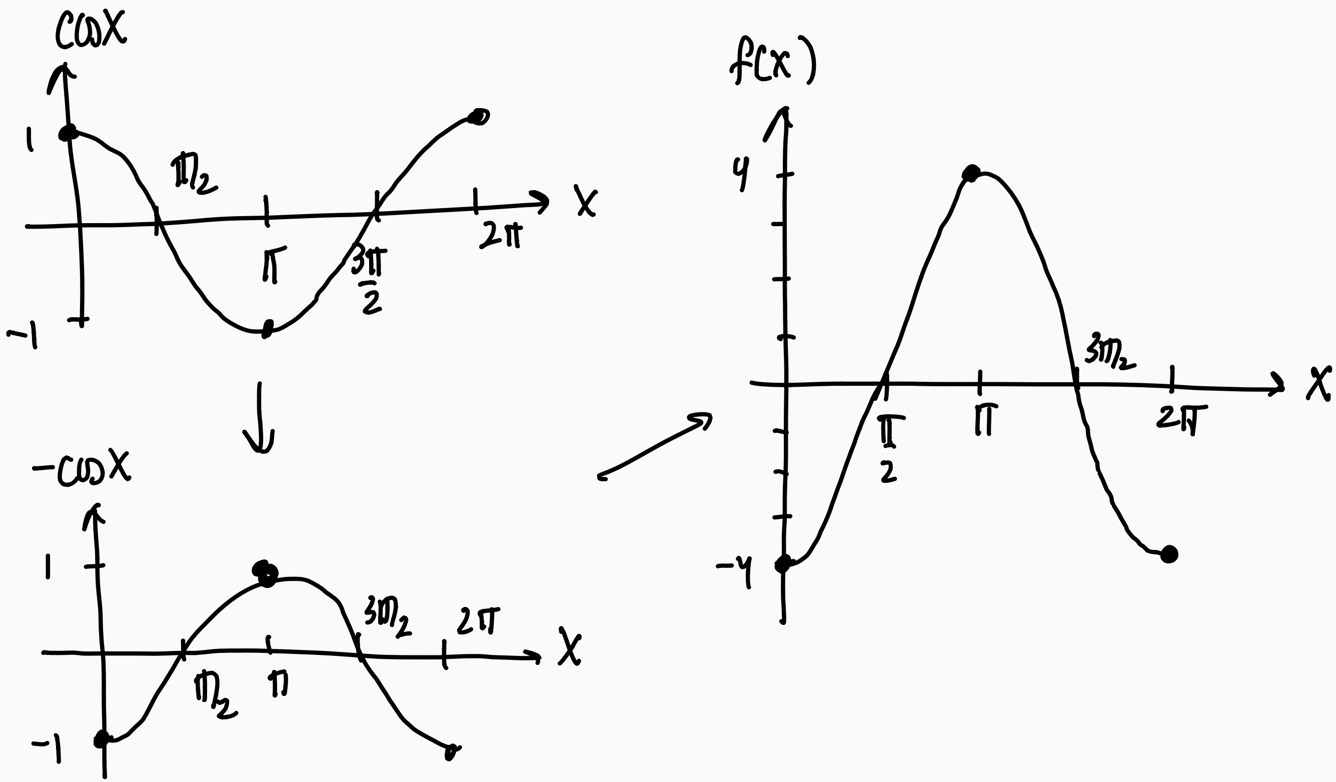
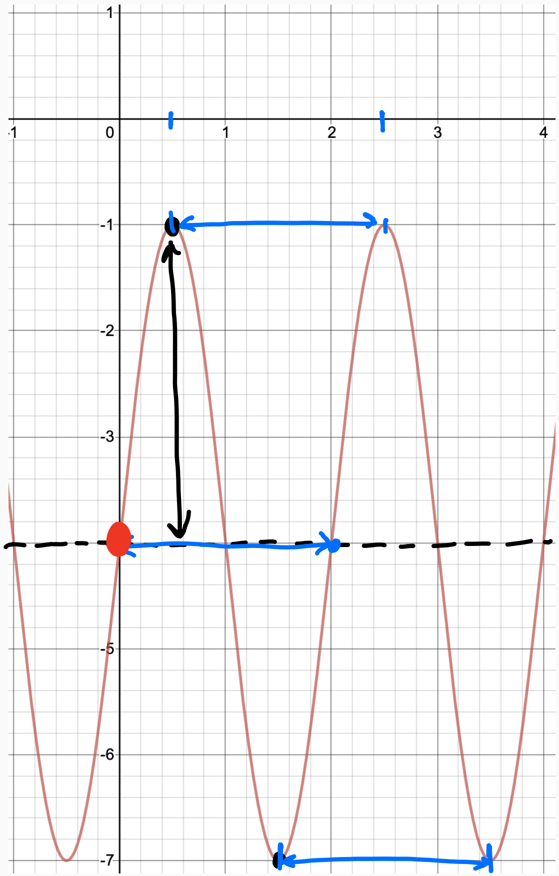
Example 1. Sketch the graph of $f(x) = \underbrace{-4}\cos x$.

vertical reflection

vertical stretch by 4 (affect amplitude)



Example 2. Determine the midline, amplitude, and period, then find a formula for the function.



$$\text{midline} = \frac{\text{largest } y \text{ value} + \text{smallest } y \text{ value}}{2}$$

$$= \frac{-1 + -7}{2} = \frac{-8}{2} = -4$$

$$\text{amplitude} = \text{largest } y \text{ value} - \text{midline}$$

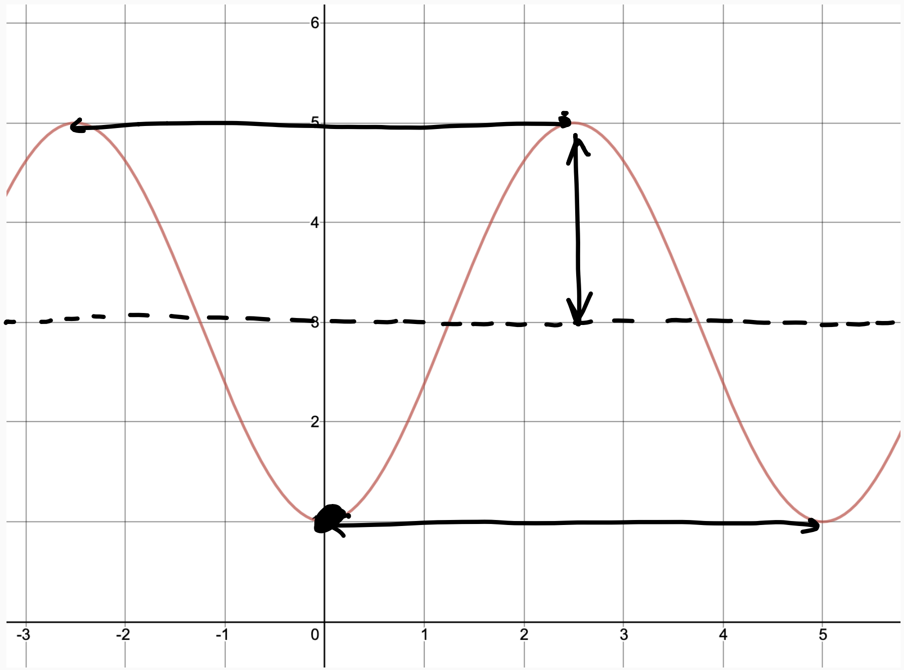
$$= -1 - (-4) = 3$$

$$\text{period} = 2 \leftarrow \quad \text{period} = \frac{2\pi}{b} \rightarrow \quad 2 = \frac{2\pi}{b} \rightarrow b = \frac{2\pi}{2} = \pi$$

$$y = A\sin(bx) + k$$

$$\boxed{y = 3\sin(\pi x) - 4}$$

Example 3. Determine the midline, amplitude, and period, then find a formula for the function.



Try on your own for a couple minutes

$$\text{midline} = \frac{5+1}{2} = 3$$

$$\text{amplitude} = 5-3=2$$

$$\text{period} = 5$$

$$y = -A\cos(bx)+k$$

$$5 = \frac{2\pi}{b} \Rightarrow b = \frac{2\pi}{5}$$

$$\boxed{y = -2\cos\left(\frac{2\pi}{5}x\right)+3}$$

Example 4. Find the amplitude, period, horizontal shift, and midline for the following equations:

(a) $y = 2\sin(3x - 21) + 4$

$= 2\sin(3(x-7))+4$

amplitude $= 2$

period $= \frac{2\pi}{3}$

horizontal shift $=$ right 7

midline $= 4$

(b) $y = \sin\left(\frac{\pi}{6}x + \pi\right) - 3$

$= \sin\left(\frac{\pi}{6}(x+6)\right) - 3$

$\frac{\pi}{\frac{\pi}{6}} = \pi \cdot \frac{6}{\pi} = 6$

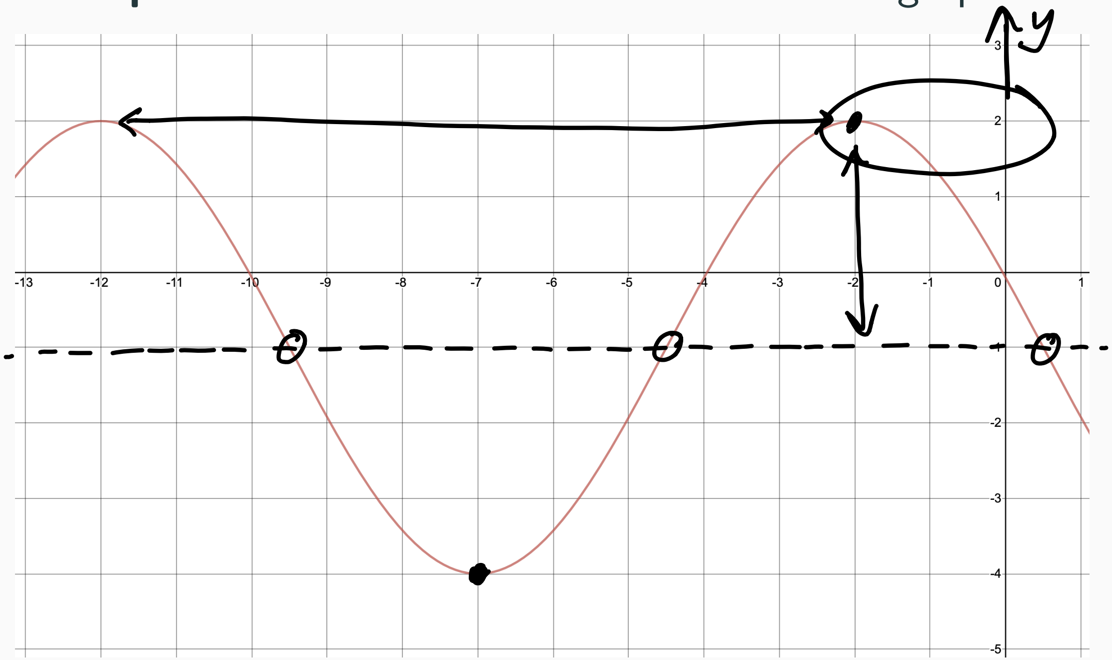
amplitude $= 1$

period $= \frac{2\pi}{\frac{\pi}{6}} = 2\cancel{\pi} \cdot \frac{6}{\cancel{\pi}} = 12$

horizontal shift $=$ left 6

midline $= -3$

Example 5. Find the formula for the function graphed below:



midline $= \frac{2+-4}{2} = \frac{-2}{2}$
$= -1$

amplitude $= 2-(-1)$
$= 3$

period $= 10$

peak $\to$ left 2 units

$10 = \frac{2\pi}{b}$

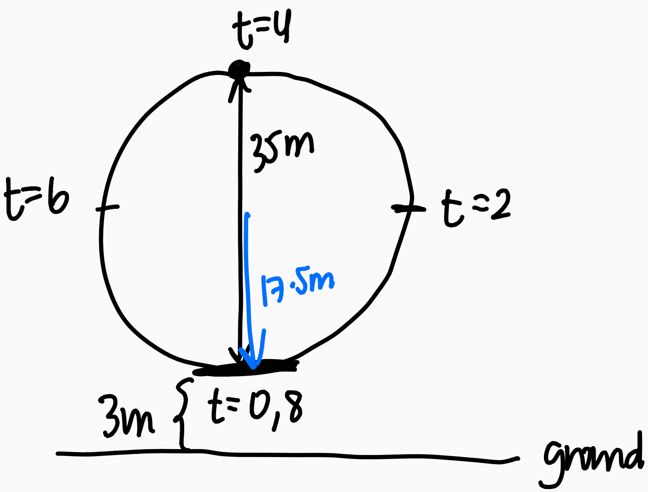
$b = \frac{2\pi}{10} = \frac{\pi}{5}$

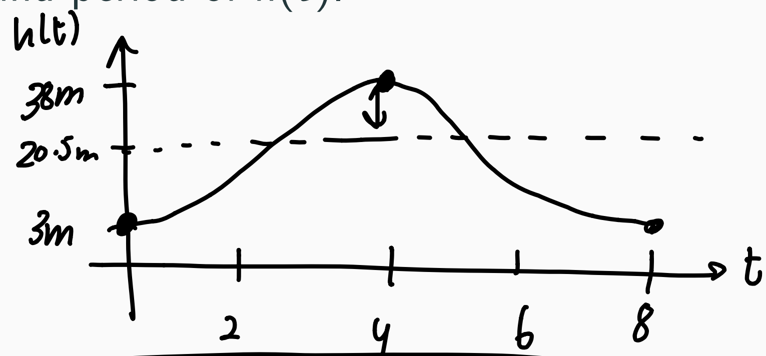
$y = A\cos(b(x-h))+k$

$= \boxed{3\cos\left(\frac{\pi}{5}(x+2)\right)-1}$

Example 6. A Ferris wheel is 35 meters in diameter and boarded from a platform that is 3m above the ground. The six o'clock position on the Ferris wheel is level with the loading platform. The wheel completes 1 full revolution in 8 minutes. The function $h(t)$ gives your height in meters above the ground t minutes after the wheel begins to turn.

(a) Find the amplitude, midline, and period of $h(t)$.





$$\text{midline} = \frac{38+3}{2} = \frac{41}{2} = 20.5\text{m}$$

$$\text{amplitude} = 38 - 20.5 = 17.5\text{m} \quad \text{(radius)}$$

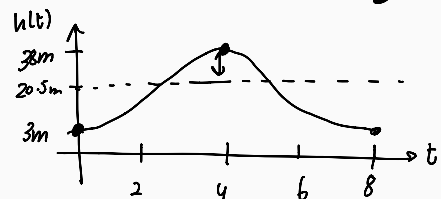
$$\text{period} = 8 \text{ min}$$

Example 6. A Ferris wheel is 35 meters in diameter and boarded from a platform that is 3m above the ground. The six o'clock position on the Ferris wheel is level with the loading platform. The wheel completes 1 full revolution in 8 minutes. The function $h(t)$ gives your height in meters above the ground t minutes after the wheel begins to turn.

$\frac{2\pi}{b} = 8$

$b = \frac{2\pi}{8}$

(b) Find a formula for the height of the function $h(t)$.



$y = -A\cos(bx) + k$

$$\boxed{h(t) = -17.5\cos\left(\frac{\pi}{4}t\right) + 20.5}$$

midline $= \frac{38+3}{2} = \frac{41}{2} = 20.5$m

amplitude $= \cancel{3}8 - 20.5 = 17.5$m (radius)

period $= 8$ min

(c) How high are you off the ground after 4 minutes?

from sketch 38m

another way $h(4) = -17.5\cos\left(\frac{\pi}{4}\cdot 4\right) + 20.5 = -17.5\cos(\pi) + 20.5$

$= 17.5 + 20.5 = 38$m

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Example 7. Evaluate the following expressions, giving the answer in radians: range $\sin^{-1}$: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ range $\cos^{-1}$: $[0, \pi]$

(a) $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

What angle in QI or ~~QIV~~ gives $\sin\theta = \frac{\sqrt{2}}{2}$?

(b) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$

~~QI~~ or QII $\cos\theta = -\frac{\sqrt{2}}{2}$

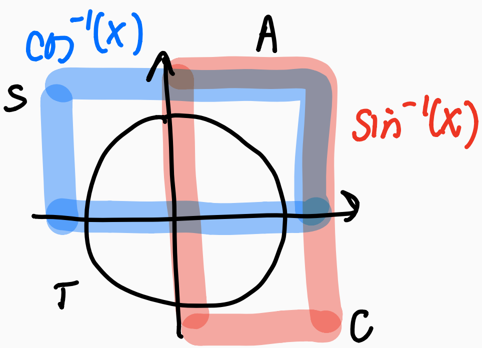
(c) $\sin^{-1}\left(-\frac{\sqrt{1}}{2}\right)$ $-\frac{\pi}{6}$

~~QI~~ QIV $\sin\theta = -\frac{1}{2}$

(d) $\cos^{-1}\left(\frac{\sqrt{1}}{2}\right) = \frac{\pi}{3}$

QI or ~~QII~~

$\cos\theta = \frac{1}{2}$

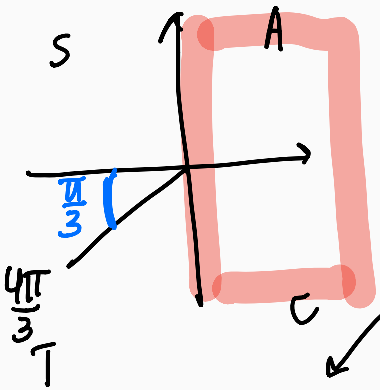


Example 8. Evaluate the following expressions:

(a) $\sin^{-1}\left(\underbrace{\cos\left(\frac{4\pi}{3}\right)}\right)$

$\cos\left(\frac{4\pi}{3}\right)$

$\frac{4\pi}{3}\cdot\frac{180^\circ}{\pi} = 240^\circ$



$\cos\left(\frac{4\pi}{3}\right) = -\cos\left(\frac{\pi}{3}\right)$

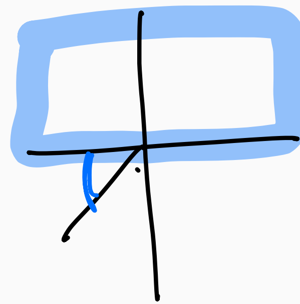
$= -\frac{1}{2}$

$\sin^{-1}\left(\frac{-1}{2}\right) = -\frac{\pi}{6}$

QIV $\quad \sin\theta = \frac{-1}{2}$

(b) $\cos^{-1}\left(\sin\left(\frac{5\pi}{4}\right)\right)$

$\frac{5\pi}{4}\cdot\frac{180^\circ}{\pi} = 225^\circ$



$\sin\left(\frac{5\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right)$

$= -\frac{\sqrt{2}}{2}$

$\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$

QII

$\cos\theta = -\frac{\sqrt{2}}{2}$

$12\cos^2(x) + 11\sin x - 14 = 0$ $\quad 0 \leq x < 2\pi$

rewrite in terms of sin

$\cos^2 x + \sin^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

$12(1 - \sin^2 x) + 11\sin x - 14 = 0$

$12 - 12\sin^2 x + 11\sin x - 14 = 0$

$-12\sin^2 x + 11\sin x - 2 = 0$ $\quad u = \sin x$

$-12u^2 + 11u - 2 = 0$

$12u^2 - 11u + 2 = 0$ quadratic

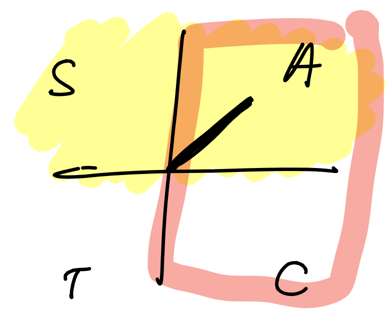
$u = \dfrac{11 \pm \sqrt{11^2 - 4(12)(2)}}{2(12)} = \dfrac{11 \pm \sqrt{121 - 96}}{24} = \dfrac{11 \pm \sqrt{25}}{24}$

$= \dfrac{11 \pm 5}{24}$ $\quad \dfrac{16}{24} = \dfrac{2}{3}$ $\quad \dfrac{6}{24} = \dfrac{1}{4}$

$u = \dfrac{2}{3}$ $\quad u = \dfrac{1}{4}$

$\sin x = \dfrac{2}{3}$ $\quad \sin x = \dfrac{1}{4}$

$x = \sin^{-1}\left(\dfrac{2}{3}\right)$ $\quad x = \sin^{-1}\left(\dfrac{1}{4}\right)$



$= \_\_\_\_$ angle in QI $\quad \_\_\_\_ + \dfrac{\pi}{2}$

$= \_\_\_\_$ angle QI $\quad \_\_\_\_ + \dfrac{\pi}{2}$