Math 1 - Summer 2020 - July 7

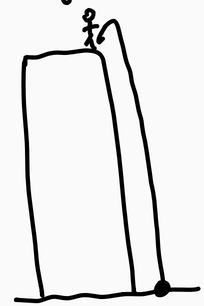
Important Concepts

- Quadratic equations
  - Vertex/transformation form $f(x) = a(x-h)^2 + k$
- Polynomials
  - Long run behavior comes from degree and leading coefficient
  - Short run behavior comes from intercepts and multiplicities
- Rational Functions
  - Vertical asymptotes come from the denominator of the function
  - Horizontal asymptotes depend on degree of numerator and denominator

Example 1. A person on Planet A kicks a ball from the top of a building, and its height after t seconds is given by $h(t) = -3t^2 + 18t + 9$.

$a = -3 \quad b = 18$

$t=0$



(a) What was the height of the building?

$t=0 \qquad h(0) = -3(0)^2 + 18(0) + 9 = 9$

(b) What is the $\underbrace{\text{maximum}}_{\text{vertex}}$ height the ball reaches?

$h = \frac{-18}{2(-3)} = \frac{-18}{-6} = 3$ seconds

time

not the same as h(t)

$k = h(3) = -3(3)^2 + 18(3) + 9$
$= -27 + 54 + 9$
$= 27 + 9$
$= 36$ height

(c) When does the ball hit the ground?

$h(t) = 0$

$-3t^2 + 18t + 9 = 0$

$t^2 - 6t - 3 = 0$

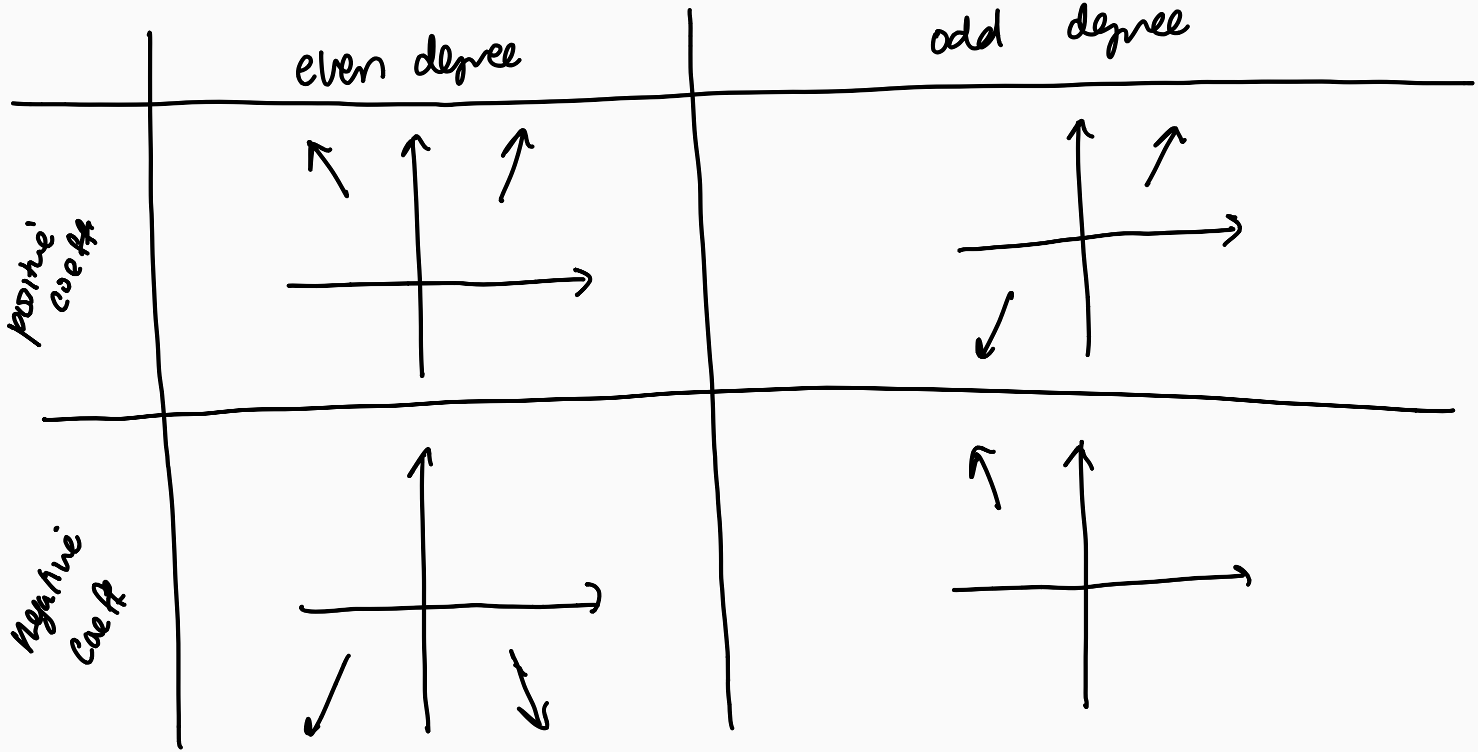
$t = \frac{6 \pm \sqrt{6^2 - 4(1)(-3)}}{2(1)} = \frac{6 \pm \sqrt{48}}{2}$

$= \frac{6 \pm 4\sqrt{3}}{2} = 3 \pm 2\sqrt{3}$

$3 + 2\sqrt{3}$ seconds

Long Run Behavior of a Polynomial

- degree of polynomial (even or odd)
- leading coefficient (positive or negative)



Example 2. Using long run behavior, match the function with its graph.

(a) $f(x) = x^3 - 4x$ C

odd degree

+ coeff ↙ ↗

(b) $g(x) = -2x^4 - x^3 + 3x^2$ D

even degree

− coeff ↙ ↘

(c) $h(x) = -x^3 - 5x$ B

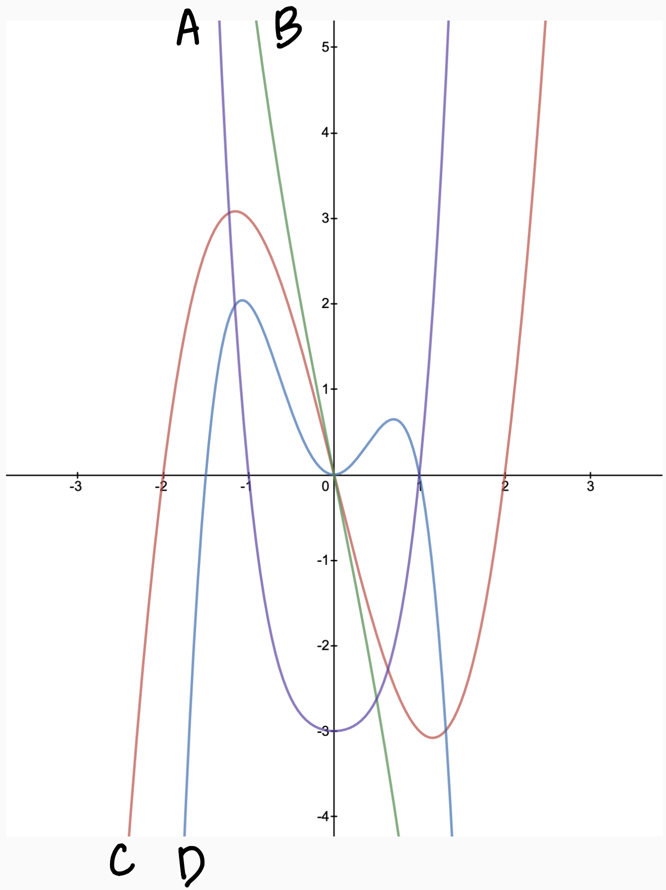
odd degree ↖

− coeff ↘

(d) $k(x) = 2x^4 + x^2 - 3$ A

even degree ↖ ↗

+ coeff



Example 3. Given $P(x) = 3(x-1)^2(x+1)^1(x+2)^1$ answer the following:

(a) What is the leading term of $P(x)$? $3x^4$

+ coeff
even degree

$\nwarrow \quad \uparrow$

(b) What is the degree of $P(x)$? 4

(c) As $x \to \infty$, $P(x) \to$ $\underline{\infty}$

(d) As $x \to -\infty$, $P(x) \to$ $\underline{\infty}$

(e) What are the coordinates of the x-intercepts? What are their multiplicities?

x mult. are the exponents

$3(x-1)^2(x+1)^1(x+2)^1 = 0$

$(x-1)^2 = 0$ $\quad$ $x+1=0$ $\quad$ $x+2=0$

$x=1$ $\quad$ $x=-1$ $\quad$ $x=-2$

$\boxed{(1,0) \text{ mult } 2 \quad (-1,0) \text{ mult } 1 \quad (-2,0) \text{ mult } 1}$

(f) What are the P-intercepts?

$x=0$ $\quad$ $P(0) = 3(0-1)^2(0+1)(0+2) = 3\cdot1\cdot1\cdot2 = 6$

$\boxed{(0,6)}$

(g) Sketch the graph of $P(x)$ (next slide).

Example 3. Given $P(x) = 3(x-1)^2(x+1)(x+2)$ answer the following:

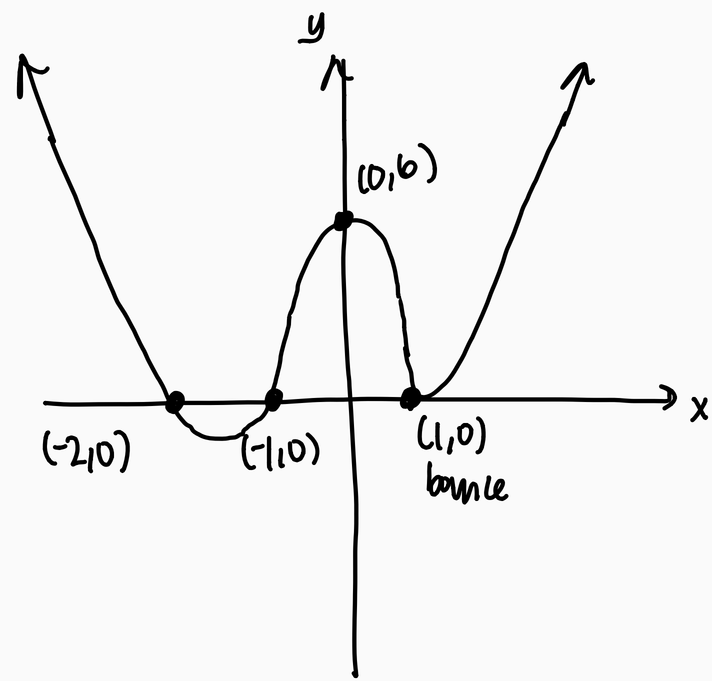
(g) Sketch the graph of $P(x)$.

end behavior $\nwarrow$ $\nearrow$

x-int $(1,0)$, $(-1,0)$, $(-2,0)$

mult 2 mult 1 mult 1

y-int $(0,6)$



even multiplicity

Example 4. Given $P(x) = 2(x-1)^{1}(x+1)^2(x+2)^2$ answer the following:

Try on your own for a couple min.

(a) What is the leading term of $P(x)$? $2x^5$

(b) What is the degree of $P(x)$? 5

+ coeff
odd degree

↑

(c) As $x \to \infty$, $P(x) \to$ $\underline{\infty}$

(d) As $x \to -\infty$, $P(x) \to$ $\underline{-\infty}$

✓

(e) What are the coordinates of the x-intercepts? What are their multiplicities?

$$2(x-1)^{1}(x+1)^{2}(x+2)^{2}=0$$

$x-1=0$	$(x+1)^2=0$	$(x+2)^2=0$
$x=1$	$x=-1$	$x=-2$
$(1,0)$	$(-1,0)$	$(-2,0)$
mult 1	mult 2	mult 2

(f) What are the P-intercepts?

$x=0$

$$P(0) = 2(0-1)(0+1)^2(0+2)^2 = 2(-1)(1)(4) = -8 \quad \boxed{(0,-8)}$$

(g) Sketch the graph of $P(x)$ (next slide).

Example 4. Given $P(x) = 2(x-1)(x+1)^2(x+2)^2$ answer the following:

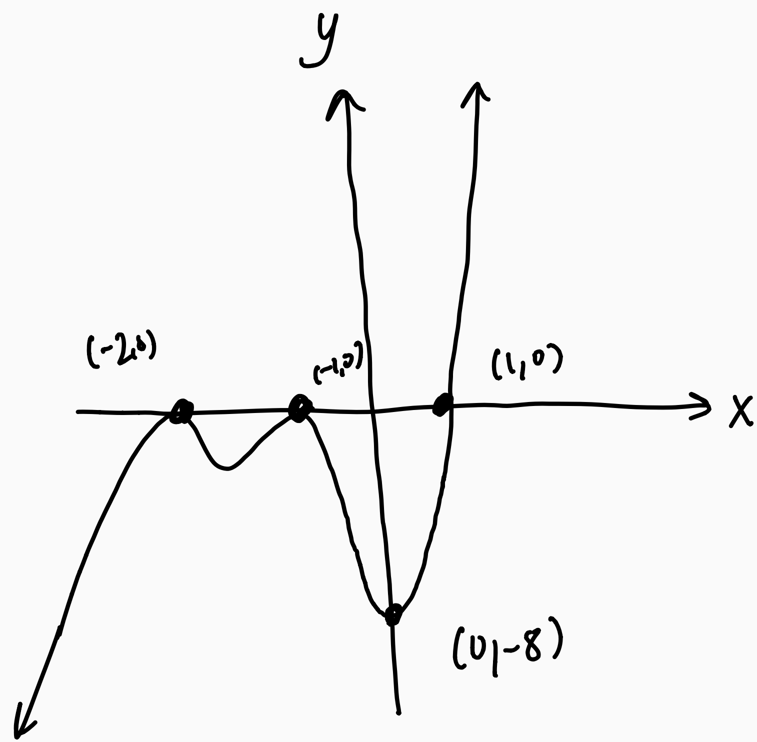
(g) Sketch the graph of $P(x)$.

long run $\nearrow$

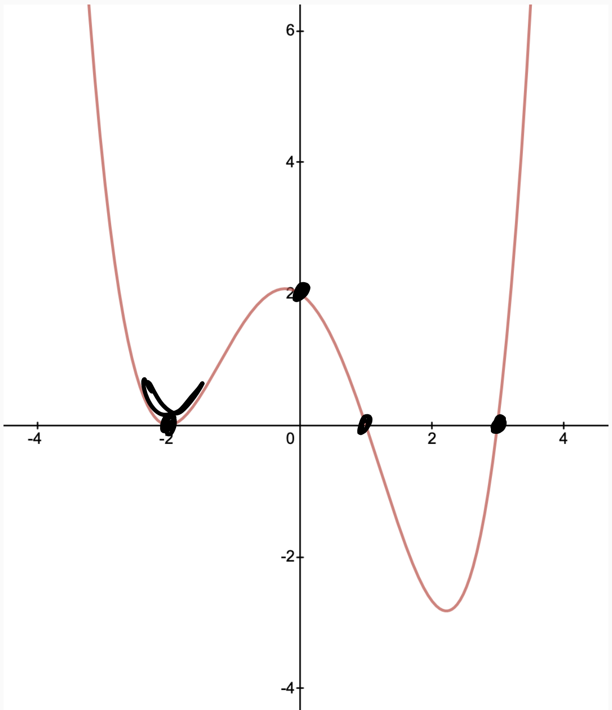
$\swarrow$

$(1,0)$ $\underbrace{(-1,0) \quad (-2,0)}_{\text{mult 2 bounce}}$

$(0,-8)$



Example 5. Write the equation of the function whose graph is given below.



long run behavior

↖ ↗

even degree

pos coeff

intercepts

x-int

(-2,0) (1,0) (3,0)

mult 2

$p(x) = a(x+2)^2(x-1)^1(x-3)^1$

y-int: (0,2)

$p(0) = a(0+2)^2(0-1)(0-3) = 2$

$a(4)(-1)(-3) = 2$

$12a = 2$

$a = \frac{1}{6}$

$$\boxed{p(x) = \frac{1}{6}(x+2)^2(x-1)(x-3)}$$

Example 6. Write the equation of the function whose graph is given below.

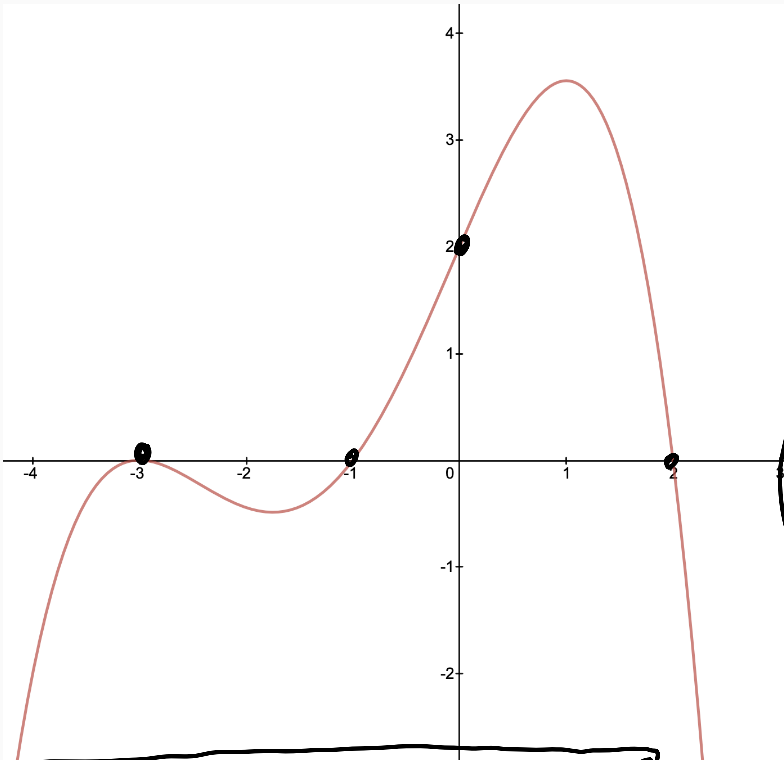
long run behavior

even degree

negative coeff

x-int $(-3,0)$ $(-1,0)$ $(2,0)$

mult 2



$$p(x) = a(x+3)^2(x+1)(x-2)$$

y-int $(0,2)$

$$p(0) = a(0+3)^2(0+1)(0-2) = 2$$

$$a(9)(1)(-2) = 2$$

$$-18a = 2$$

$$a = \frac{-2}{18}$$

$$a = \frac{-1}{9}$$

$$\boxed{p(x) = \frac{-1}{9}(x+3)^2(x+1)(x-2)}$$

Horizontal Asymptotes of a Rational Function

Example 7. Given $Q(x) = \dfrac{(x-1)(x+1)}{(x-2)^2(x+2)}$, answer the following:

(a) What are the coordinates of the x-intercepts?

(b) What are the coordinates of the Q-intercepts?

(c) What are the vertical asymptotes?

(d) What are the horizontal asymptotes?

(e) As $x \to \infty$, $Q(x) \to$ ______

(f) As $x \to -\infty$, $Q(x) \to$ ______

(g) Sketch the graph of $Q(x)$ (next slide)

Example 7. Given $Q(x) = \dfrac{(x-1)(x+1)}{(x-2)^2(x+2)}$, answer the following:

(g) Sketch the graph of $Q(x)$ (next slide)

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Example 8. Given $Q(x) = \dfrac{3(x+3)(x-1)}{(x-2)^2}$, answer the following:

(a) What are the coordinates of the x-intercepts?

(b) What are the coordinates of the Q-intercepts?

(c) What are the vertical asymptotes?

(d) What are the horizontal asymptotes?

(e) As $x \to \infty$, $Q(x) \to$ ______

(f) As $x \to -\infty$, $Q(x) \to$ ______

(g) Sketch the graph of $Q(x)$ (next slide)

Example 8. Given $Q(x) = \dfrac{3(x+3)(x-1)}{(x-2)^2}$, answer the following:

(g) Sketch the graph of $Q(x)$ (next slide)