Math 1 - Summer 2020 - June 25

Office Hours: Office hours will be held Wednesdays 5-6pm PT and Thursdays 2-3pm PT. Links will be posted on CCLE. I am also available by appointment, email me at victoriakala@ucla.edu.

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Important Concepts

- Piecewise functions
- Composition of functions
- The **average rate of change** of a function f on the interval $[a, b]$ is

$$\frac{f(b) - f(a)}{b - a}$$

- The equation of a line with slope m and y-intercept $(0, b)$ is

$$y = mx + b$$

  The equation of a line with slope m passing through the point (x_1, y_1) is

$$y - y_1 = m(x - x_1)$$

- If two lines are **parallel**, they have the same slope.

Important Concepts

- If two lines are **perpendicular**, then

$$m_2 = -\frac{1}{m_1}$$

  where m_1, m_2 are the slopes of the lines.

- If $f(x)$ is a function, then

$$g(x) = af(b(x-h)) + k$$

  is a **transformation** of f that is

  1. Reflected vertically (about the x-axis) if $a < 0$
  2. Stretched vertically by a factor of $|a|$ if $|a| > 1$, compressed if $|a| < 1$
  3. Reflected horizontally (about the y-axis) is $b < 0$
  4. Compressed horizontally by a factor of $|b|$ if $|b| > 1$, stretched if $|b| < 1$
  5. Shifted right by h units if $h > 0$, left if $h < 0$
  6. Shifted up k units if $k > 0$, down if $k < 0$

* left → right
* outside : vertical
* inside : horizontal

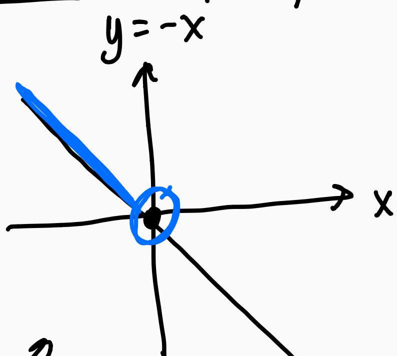
Example 1. Sketch a graph of

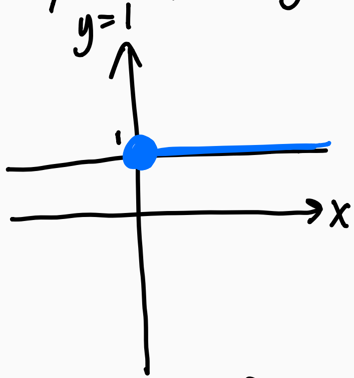
$$f(x) = \begin{cases} -x, & \boxed{x < 0} \\ 1, & x \geq 0 \end{cases}$$

piecewise function

ignore at beginning

Step 1 Graph $y=-x$ and $y=1$ separately.



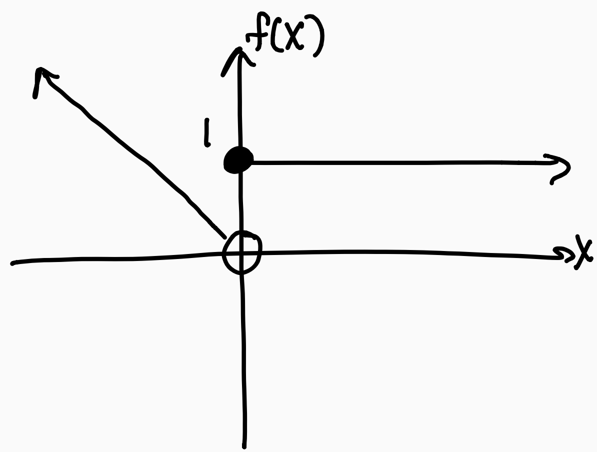


Step 2. Apply the domain

$x<0$

$x \geq 0$

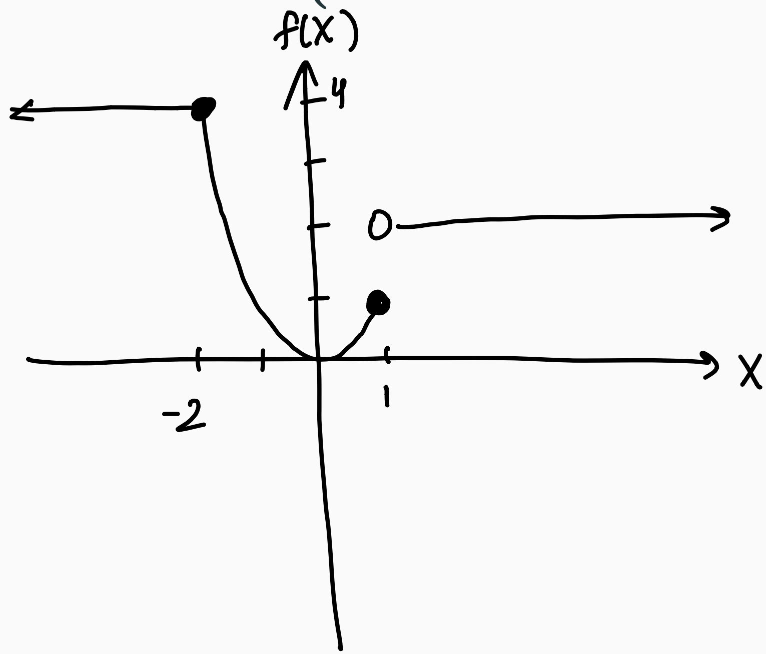
Step 3 Put together



Example 2. Sketch a graph of

Try on your own for
a couple minutes

$$f(x) = \begin{cases} 4, & x < -2 \\ x^2, & -2 \le x \le 1 \\ 2, & x > 1 \end{cases}$$



composition

Example 3. Let $f(x) = \dfrac{x}{x+1}$ and $g(x) = 2x - 1$. Evaluate the following:

(a) $f(\underbrace{g(4)}_{})$

inside → out

$g(4) = 2(4) - 1 = 8 - 1 = 7$

$f(g(4)) = f(7) = \dfrac{7}{7+1} = \dfrac{7}{8}$

(b) $g(f(1))$ try this one on your own

$f(1) = \dfrac{1}{1+1} = \dfrac{1}{2}$

$g(f(1)) = g\left(\frac{1}{2}\right) = \cancel{2}\left(\frac{1}{\cancel{2}}\right) - 1 = 1 - 1 = 0$

Example 4. Find the average rate of change of $\boxed{f(x) = x^2}$ on the following intervals:

Avg. Rate of Change of f over interval [a,b]

$$\frac{f(b)-f(a)}{b-a}$$

Slope: $\frac{y_2-y_1}{x_2-x_1}$

(a) $[\underset{a}{1}, \underset{b}{2}]$ $\frac{f(2)-f(1)}{2-1} = \frac{4-1}{2-1} = \frac{3}{1} = 3$

$f(2) = 2^2 = 4$

$f(1) = 1^2 = 1$

(b) $[\underset{a}{1}, \underbrace{1+h}_{b}]$ $\frac{f(1+h)-f(1)}{\cancel{1}+h-\cancel{1}}$

$f(1+h) = (1+h)^2 = (1+h)(1+h) = 1+h+h+h^2 = 1+2h+h^2$

$f(1) = 1$

$$\frac{\cancel{1}+2h+h^2-\cancel{1}}{h} = \frac{2h+h^2}{h} = \frac{\cancel{h}(2+h)}{\cancel{h}} = 2+h$$

Example 5. A linear function $f(x)$ passes through the point $f(\overset{x}{-2}) = \overset{y}{-1}$ with a slope of $\frac{1}{3}$.

$$\boxed{\begin{gathered} y = mx + b \\ y - y_1 = m(x - x_1) \end{gathered}}$$

(a) Write the equation for $f(x)$.

$m = \frac{1}{3}$ slope

$(\underset{x_1}{-2}, \underset{y_1}{-1})$ point

point-slope

$y - (-1) = \frac{1}{3}(x - (-2))$

$\boxed{y + 1 = \frac{1}{3}(x + 2)}$

$y + 1 = \frac{1}{3}x + \frac{2}{3}$

$\frac{2}{3} - \frac{3}{3}$

$y = \frac{1}{3}x + \frac{2}{3} - 1$

$\boxed{f(x) = \frac{1}{3}x - \frac{1}{3}}$

(b) Write the equation of a line $g(x)$ that is perpendicular to $f(x)$ and passes through the point $(0, 2)$.

$m_2 = \frac{-1}{m_1}$

$(0, 2) \leftarrow$ y-int because $x = 0$

$m = \frac{-1}{\frac{1}{3}} = -1 \cdot \frac{3}{1} = -3$

$y = -3x + 2$

$\boxed{g(x) = -3x + 2}$

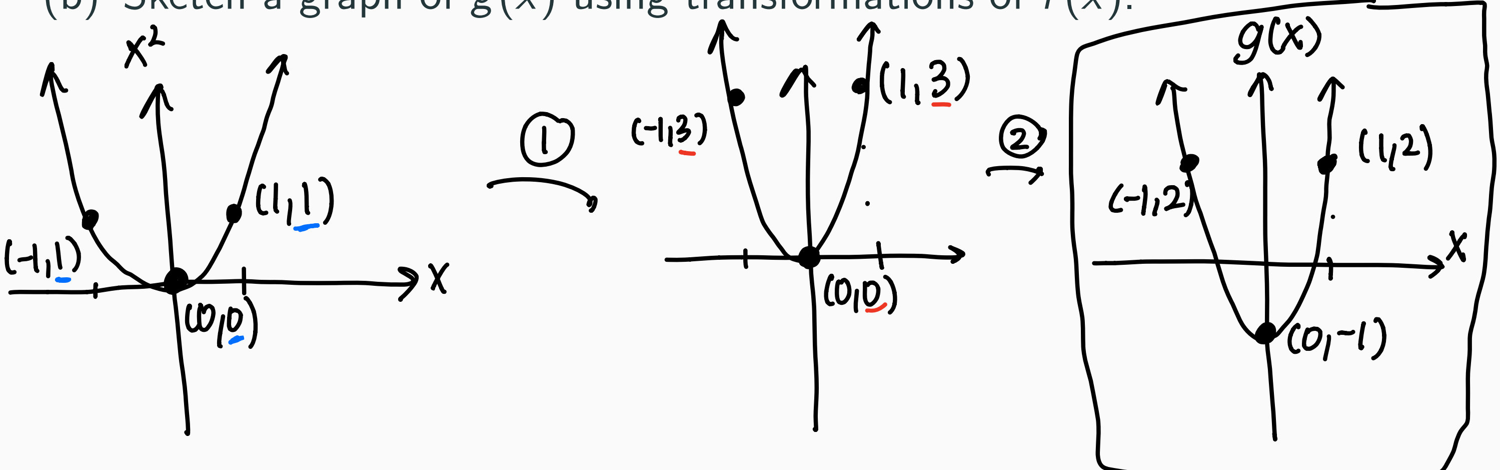
Example 6. Consider the function $g(x) = 3x^2 - 1$, which is a transformation of the function $f(x) = x^2$. $\longrightarrow$ left to right

inside: $(3x)^2$ outside: $3 \cdot x^2$ inside: $(x-1)^2$ outside: $x^2 - 1$

(a) Describe the transformations used to obtain $g(x)$ from $f(x)$.

1. vertical stretch by 3 (multiply y values by 3)
2. vertical shift down 1 unit (subtract 1 from y values)

(b) Sketch a graph of $g(x)$ using transformations of $f(x)$.



Example 7. Consider the function $g(x) = -1\sqrt{x+3}$, which is a transformation of the function $f(x) = \sqrt{x}$.

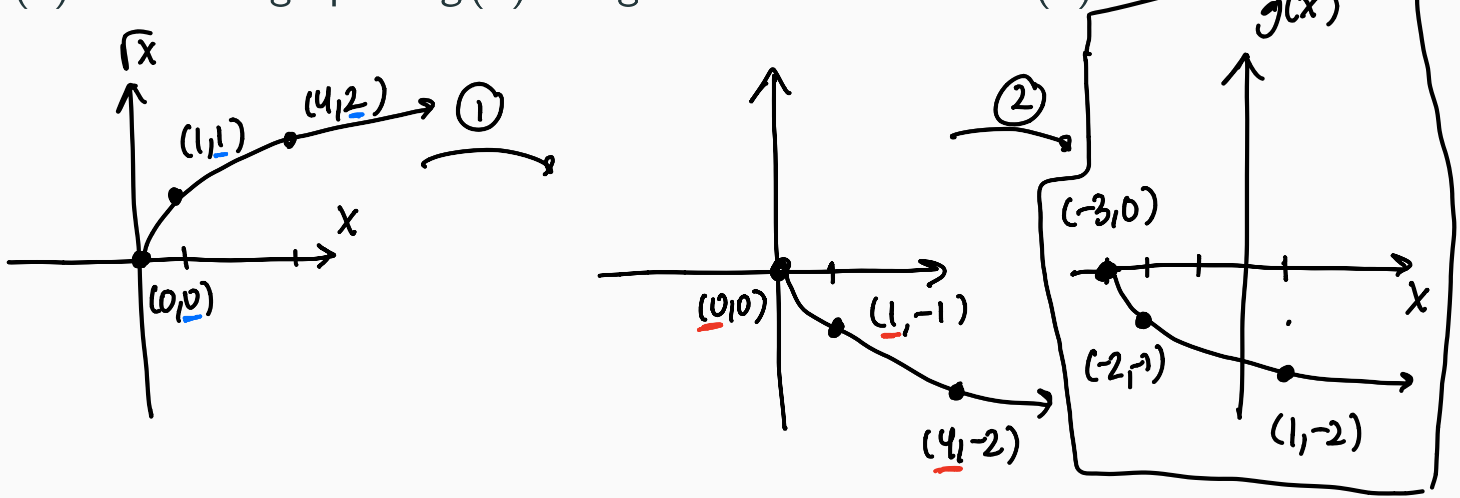
inside: $\sqrt{-1x}$ outside: $-1\sqrt{x}$ inside: $\sqrt{x+3}$ outside: $\sqrt{x}+3$

(a) Describe the transformations used to obtain $g(x)$ from $f(x)$.

① vertical reflection (multiply y values by -1)

② horizontal shift left 3 units (subtract 3 from x values)

(b) Sketch a graph of $g(x)$ using transformations of $f(x)$.



$y = af(bx)$

vertical stretch / compression / reflections

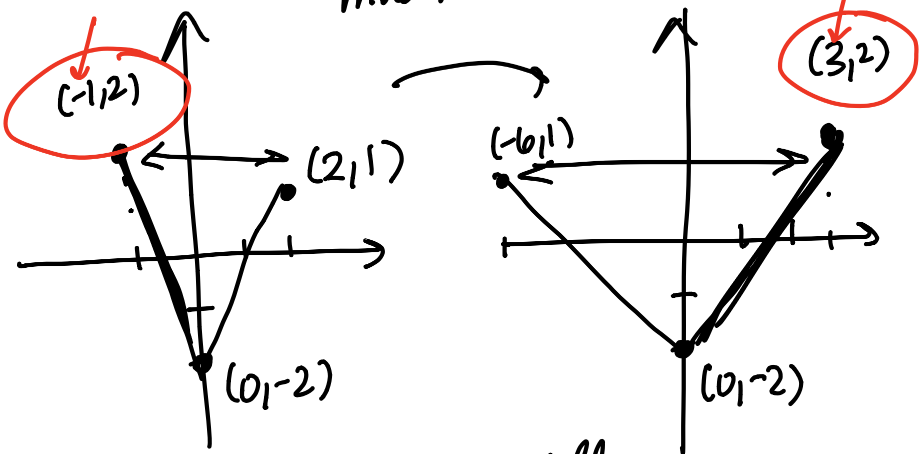
horizontal compression / stretch / reflection

$a = 1$

$b = -\frac{1}{3}$

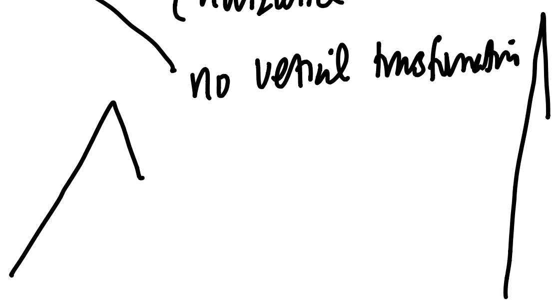
HW 1 #25

multiplying x values by -3



b { flip horizontally
horizontal stretch

no vertical transformation



Inverse Function

- (x, y) is on graph of $f(x)$ & f is one-to-one $\longrightarrow$ (y, x) is on graph of $f^{-1}(x)$

e.g. $f(0) = 4$ what is $f^{-1}(4)$? $f^{-1}(4) = 0$

- ① replace $f(x) = y$
  ② swap y & x values
  ③ solve for y
  ④ replace y w/ $f^{-1}(x)$

e.g. find $f^{-1}(x)$ of $f(x) = x + 1$

① $y = x + 1$
② $x = y + 1$
③ $y = x - 1$
④ $\boxed{f^{-1}(x) = x - 1}$

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